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EFFECTIVE METHODS FOR SOLUTION OF NONLINEAR REACTOR DYNAMICS PROBLEMS USING FINITE ELEMENTS

Richard Allen Olsen

NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

Effective Methods for Solution of Nonlinear Reactor Dynamics Problems Using Finite Elements

by

Richard Allen Olsen

December 1975

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Co-Advisor: D. Salinas

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This system has been solved directly in a previous work; but because the nonlinearity $C_{ijk}\psi_j\psi_k$ was premultiplied by $[A]^{-1}$, large computer storage

was required for the small problem considered. The task of this thesis is the development of computational techniques which allow the problem to be solved for large systems. Specifically, these techniques are: (1) the treatment of the nonlinearity on the element level, (2) the compacting of the sparce matrices to include only non-zero terms, and (3) the construction of a new computer code based on the Crank-Nicolson formulation for the solution of differential equations.

To support the theory presented, test problems were solved by the original method, the linearized technique, and the Crank-Nicolson treatment. The results were analyzed and compared graphically. All three of the innovations developed in this thesis appear to be useful tools for solving nonlinear time dependent differential equations.

Effective Methods for Solution of Nonlinear Reactor Dynamics Problems Using Finite Elements

by

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December 1975

ABSTRACT

The solution of the nonlinear two-dimensional reactor dynamics equation subjected to prompt feedback conditions using the finite element technique leads to the matrix formulation $A_{ij}\dot{\psi}_j = B_{ij}\psi_j + C_{ijk}\psi_j\psi_k$ $(i,j,k = 1, \ldots, N)$. This system has been solved directly in a previous work; but because the nonlinearity $C_{ijk}\psi_j\psi_k$ was premultiplied by $[A]^{-1}$, large computer storage was required for the small problem considered. The task of this thesis is the development of computational techniques which allow the problem to be solved for large systems. Specifically, these techniques are: (1) the treatment of the nonlinearity on the element level, (2) the compacting of the sparce matrices to include only non-zero terms, and (3) the construction of a new computer code based on the Crank-Nicolson formulation for the solution of differential equations.

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I. INTRODUCTION

When temperature-dependent feedback is considered in the nuclear reactor dynamics problem, a nonlinear field equation in space and time results [Ref. 1]. For the non-homogeneous, or multi-region reactor, however, the space-dependent dynamics behavior following a nonlinear initial disturbance is no longer reachable in analytical form. Fortunately, by modeling the reactor as a system of finite regions of interest where the neutronic properties are known, the method of finite elements can be applied to yield the solutions. The fundamental concepts relating reactor behavior to the finite element formulation have been presented in 1974 [Ref. 2], but a more recent work by Nguyen and Salinas [Ref. 3] gives a thorough discussion of the complete problem. That work forms the basis and starting point for this thesis, the objective of which is to develop improved computational methods for dealing with that finite element formulation.

In Reference 3 the dynamic flux equation under prompt feedback conditions was given as

$$\frac{1}{V_m} \frac{\partial \psi_m(r,z,t)}{\partial t} = D_m \nabla^2 \psi_m + \gamma_m \Sigma_{am} \psi_m - \alpha_m K_m \Sigma_{am} \psi_m^2 \tag{1}$$

for each non-homogeneous zone "m" contained in the reactor body, where the usual symbols are employed:

$$\lambda_m = \nu\, \Sigma_{fm}/\Sigma_{am} - 1$$

$$K_m = e\, \Sigma_{fm}/\rho_m C_{pm} \quad (°\text{C/unit flux-sec})$$

$$\gamma_m = (A_m/V_m)(h_m/\rho_m C_{pm}) \quad \text{sec}^{-1}$$

$\rho\ C_p$ = heat capacity

α = reactivity temperature coefficient $°C^{-1}$

h = convection heat transfer coefficient

A/V = heat transfer area to volume of energy generation ratio

Σ_f = neutron fission cross-section

Σ_a = neutron absorption cross-section

ν = neutrons emitted per fission

e = energy produced per fission

The subscript "m" will be omitted from further discussion for simplicity.

The transformation of this general equation into a problem in finite dimensional vector space begins by constructing the following N term approximation[1]

$$\psi(\underline{x},t) \simeq \tilde{\psi}(\underline{x},t) = \sum_{j=1}^{N} \psi_j(t)G_j(\underline{x}) \tag{2}$$

where N is the number of degrees of freedom, or nodes, in the space, and the G_j are the basis functions of the approximate solution space. The Galerkin method seeks to make the residual

$$R(x,t) = L\,\tilde{\psi} - f,$$

where $L\tilde{\psi} = f$ is the field equation, othogonal to each of the basis functions so that

[1] This discussion leading to the establishment of the matrix equations is essentially an abridgement of the development given in reference 3.

$$\int_S G_i(\underline{x})R(\underline{x},t)\,dx = 0 \qquad i = 1, 2, \ldots , N . \tag{3}$$

For the nuclear reactor dynamics problem of Eq. (1), the residual becomes

$$R(\underline{x},t) \;=\; \frac{\partial \psi^2}{\partial t} - V\, D\nabla^2\tilde{\psi} - V\lambda\Sigma_a\tilde{\psi} + w\Sigma_a\tilde{\psi}^2 \tag{4}$$

with $V\alpha K\Sigma_a = w$. Putting this expression into Eq. (3) and integrating by parts (a distinct advantage of Galerkin), the following coefficients emerge

$$A_{IJ} = \iint_S G_I G_K \; rdrdz \tag{5a}$$

$$B_{IJ} = \iint_S \left[\frac{\partial G_J}{\partial r} \cdot \frac{\partial G_I}{\partial r} + \frac{\partial G_J}{\partial z} \cdot \frac{\partial G_I}{\partial z} \right] rdrdz \tag{5b}$$

$$C_{IJK} = \iint_S G_I G_J G_K \; rdrdz \tag{5c}$$

$$I,J,K = 1, \ldots, N$$

in cylindrical coordinates where $dS = rdrdz$. This allows Eq. (1) to be written as follows for the case of uniform neutronic properties within each reactor region m.

$$\sum_{J=1}^{N} A_{IJ}\dot{\psi}_J = - V_I D_I \sum_{J=1}^{N} B_{IJ}\psi_J + V_I\lambda_I\Sigma_{aI} \sum_{J=1}^{N} A_{IJ}\psi_J - w_I \sum_{J=1}^{N}\sum_{K=1}^{N} C_{IJK}\psi_J\psi_K \qquad I = 1,2, \ldots N \tag{6}$$

This becomes, upon combining constants, in Einstein summation convention, where there is no sum on an underlined repeated index,

$$A_{IJ}\,\dot{\psi}_J = AB_{IJ}\,\psi_J - w_{\underline{I}}C_{\underline{I}JK}\,\psi_J\psi_K \tag{7a}$$

$$\dot{\psi}_i = f_i(\psi_j,t) \qquad\qquad I,J,K, = 1, \ldots, N$$

With boundary conditions, the system is now well posed and ready for solution. The equation-solver must be chosen with care, however, because this system of ordinary differential equations is both stiff and non-linear. "Stiff" is used here and throughout this thesis to denote a system which gives a large response to a small stimulus; in this case, the nature of the reactor dynamics problem predicts a large change in flux over a small change in time.[2] Current experience suggests Gear's predictor-corrector method [Ref. 4] as written in the computer programming code DVOGER [Ref. 5]. It requires only 1) the locus (value) of the points at a given time, 2) a routine to evaluate the instantaneous derivatives at any time and 3) a routine to evaluate the Jacobian ($J = \partial f_i(\psi,t)/\partial\psi_j$). Based on this information, the program assumes a time step, which may be quite small (the minimum size being a function of the particular computer), and tries to fit a trial (predicted) solution for that time interval. The corrected (integrated) solution is then obtained by iteration until convergence is attained. If the predicted and corrected solutions fail to match within a specified error criterion, the time step is decreased and the process repeated. On the other hand, if "excessive" accuracy is attained, the next attempted time step will be increased. Gear's method in DVOGER is well suited to stiff non-linear systems. Also contained in DVOGER is the Adam's

---

[2] This definition is somewhat broader in scope than that used by many other authors.

method which parallels that of Gear except that the Jacobian is not computed. Lacking the additional information about the rate of change, the Adam's method must proceed much more cautiously with stiff systems and hence marches forward with exceedingly small time steps. Present experience confirms the caveat advice given by DVOGER that the Adam's method is not intended for "stiff" cases.

The difficulty that arises from acquiring solutions to Eq. (7), then, should not, and, in light of the present experience, does not result from DVOGER or any other acceptable equation solver, but is, rather, a function of matrix size, vis a vis the number of nodal points in the finite element approximation. The finite element modeling of large reactors or the close scrutiny of small ones is severely restricted, therefore, unless the number of nodal points can be raised significantly. As an example, the test case considered in this thesis consisted of only 38 nodes but, when processed directly from Eq. (7), required over 300K bytes of storage in the IBM 360/67 computer. This requirement came largely from the $[A]^{-1}[C_{IJK}]$ matrix alone needing a size of $4(38x38x38) =$ 219K bytes in single precision. Manipulations with this cubic were indeed quite burdensome. Since machine processing time is, other things being equal, dependent on size, the combined time and space requirements would prohibit the consideration of even moderate sized problems except on the largest computers. Additionally, the inversion of $[A]$ in Eq. (7) is indicated in order to provide an explicit value of $\dot{\psi}$ for the DVOGER routine. It is this difficulty of size, and with it processing time, to which the remainder of this thesis is devoted. Experience with the DVOGER routine is presented, and the author's equation-solver based on

a Crank-Nicolson formulation is discussed as the analysis of the size problem is developed. Table I shows clearly the influence of matrix size on computer storage requirements.

II. REDUCTION OF THE NONLINEARITY

A. GLOBAL TREATMENT OF THE NONLINEARITY

The evaluation of Eq. (7) generates the following global matrix system, where the subscripts indicate the matrix size required for N number of nodal points

$$\left[A_{NxN}\right]\left\{\dot{\psi}_{Nx1}\right\} = \left[B_{NxN}\right]\left\{\psi_{Nx1}\right\} + \left[C_{NxNxN}\right]\left\{\psi^2_{(NxN)x1}\right\} \quad (8)$$

where ψ is the time response of the flux at each nodal point and ψ^2 is the product $\psi_i \psi_j$ $i,j = 1,2, \ldots N$. Solved in this form, the major limitation of the procedure was the large N^3 size of the cubic array C, which is introduced as the result of the nonlinearity of Eq. (1). With this size requirement rapidly increasing for a finer mesh consisting of more nodal points,[3] the investigation of Reference 3 was limited to a small but practical N of 38.

B. TREATMENT ON THE ELEMENT LEVEL

The cubic form of C need not appear if it can be shown that the nonlinear portion of Eq. (8) can be broken apart into the product of two linear components such that one component can be evaluated on the element level and the element contributions then summed into global form,

---

[3]In general, for a nonlinearity of order m, a finite element problem of N nodes will generate a matirx size of N^{m+1}.

resulting in a "regular" NxN matrix C* ready for multiplication by the remaining ψ also summed to the global level. Proof that this is indeed the case, i.e.,

$$\left[C_{Nx(NxN)}\right]\left\{\psi^2_{(NxN)x1}\right\} = \left[C^*_{NxN}\right]\left\{\psi_{Nx1}\right\} \tag{9}$$

is outlined as follows:

Reference 3 established the element nonlinear term as

$$\iint_{A_e} \zeta_i(\psi_j\zeta_j)(\psi_k\zeta_k)dA = C_{ijk}\ \psi_j(t)\ \psi_k(t) \tag{10}$$

where the integral is over the element area A_e ,

$$C_{ijk} = \iint_{A_e} \zeta_i\zeta_j\zeta_k(r_e\zeta_e)drdz, \quad i,j,k,e = 1,2,3 \tag{11}$$

and ζ being the local triangular coordinates, an innovation of Felippa [Ref. 6]. The coefficients C_{ijk} form a 3x3x3 element array possessing a regularity which suggests a kind of "cubic symmetry." The evaluation of this integral results in the following expression

$$\left.\begin{aligned}
C_{111} &= \gamma^e\left[24r_1 + 6(r_2 + r_3)\right] \\
C_{112} &= \gamma^e\left[6r_1 + 4r_2 + 3r^3\right] \\
C_{113} &= \gamma^e\left[6r_1 + 2r_2 + 4r_3\right] \\
C_{122} &= \gamma^e\left[4r_1 + 6r_2 + 2r_3\right] \\
C_{123} &= \gamma^e\left[2(r_1 + r_2 + r_3)\right] \\
C_{133} &= \gamma^e\left[4r_1 + 2r_2 + 6r_3\right] \\
C_{222} &= \gamma^e\left[6r_1 + 24r_2 + 6r_3\right] \\
C_{333} &= \gamma^e\left[6r_1 + 6r_2 + 24r_3\right]
\end{aligned}\right\} \tag{12}$$

where

$$\gamma^e = \pi A_e/180$$

and

$$C_{121} = C_{112} = C_{211}$$

$$C_{321} = C_{231} = C_{213} \quad \text{etc as in Ref. 3.}$$

Assuming that ψ^2 may be broken apart as $\psi^2 = \psi \cdot \phi$, the LHS (left hand side) of Eq. (10) is rewritten as

$$\iint_{A_e} \zeta_i(\zeta_j\psi_j)(\zeta_k\phi_k)\,dA \qquad i,j,k = 1,2,3 \tag{13}$$

or

$$2\pi \iint_{A_e} \zeta_K\,[\zeta_1\zeta_2\zeta_3]\begin{bmatrix}\psi_1\\ \psi_2\\ \psi_3\end{bmatrix}[\zeta_1\zeta_2\zeta_3]\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\end{bmatrix} r\,dr\,dz \tag{14}$$

Expanding and collecting terms yields

$$\begin{aligned}2\pi \iint_{A_e} \zeta_K \Big[\psi_1 \Big\{&\phi_1(r_1\zeta_1^3 + r_2\zeta_1^2\zeta_2 + r_3\zeta_1^2\zeta_3)\\ &+ \phi_2(r_1\zeta_1^2\zeta_2 + r_2\zeta_1\zeta_2^2 + r_3\zeta_1\zeta_2\zeta_3)\\ &+ \phi_3(r_1\zeta_1^2\zeta_3 + r_2\zeta_1\zeta_2\zeta_3 + r_3\zeta_1\zeta_3^2)\Big\}\\ + \psi_2 \Big\{&\phi_1(r_1\zeta_1^2\zeta_2 + r_2\zeta_1\zeta_2^2 + r_3\zeta_1\zeta_2\zeta_3) + \phi_2(r_1\zeta_1\zeta_2^2 + r_2\zeta_2^3 + r_3\zeta_2^2\zeta_3)\end{aligned} \tag{15}$$

$$+ \phi_3(r_1\zeta_1\zeta_2\zeta_3 + r_2\zeta_2^2\zeta_3 + r_3\zeta_2\zeta_3^2)\Big\}$$

$$+ \psi_3 \Big\{\phi_1(r_1\zeta_1^2\zeta_3 + r_2\zeta_1\zeta_2\zeta_3 + r_3\zeta_1\zeta_3^2) + \phi_2(r_1\zeta_1\zeta_2\zeta_3 + r_2\zeta_2^2\zeta_3 + r_3\zeta_2\zeta_3^2)$$

$$+ \phi_3(r_1\zeta_1\zeta_3^2 + r_2\zeta_2\zeta_3^2 + r_3\zeta_3^3)\Big\}\Big] \, dr \, dz \qquad k = 1,\ 2,\ 3$$

Since the quantities ψ_i and ϕ_j $(i,j = 1,2,3)$ depend only on time, they are unaffected by the integration. Further, the expressions inside the parentheses are given a unique name d_{ijk} to fix their position in relation to a particular $\psi_i\phi_j$ for some $k = 1,2,3$. Now the above expression may be written as

$$2\pi \ \psi_i\phi_j \iint_A d_{ijk} \, dr \, dz \ . \tag{16}$$

Using the integration formula from Felippa [Ref. 6]

$$\iint_{A_e} \zeta_1^{\ell} \ \zeta_2^{m} \ \zeta_3^{n} \ dA = \frac{\ell!m!n!}{(\ell+m+n+2)!} \times 2A_e \tag{17}$$

where ℓ,m,n are the exponents of ζ in a specific d_{ijk}. The integration of (16) gives

$$(\pi A_e/180) \ \psi_i\phi_j \ f_{ijk} \tag{18}$$

and $f_{ijk} \times \pi A_e/180$ is identically equal to C_{ijk} of (12).

The indicated summation is conveniently expressed by

$$\phi_j \ f_{ijk} = \alpha_i \qquad i,j,k = 1,2,3 \tag{19}$$

which is immediately recognized as $\gamma^e \alpha_{ik} = C^*_{ik}$, the desired 3x3 element matrix. In this form it is combined into the global $[C^*_{NxN}]$, and the linearization has been accomplished. As will be shown later, it may be convenient to let $\phi = \psi_{t-\Delta t}$ when for small Δt, $\phi = \psi_{t-\Delta t} \approx \psi_t$. This approximation is useful for predictor-corrector equation solvers in order to allow construction of $[C^*_{NxN}]$ only once for each successful time step.

III. SOLUTION PROCEDURE WITH THE LINEARIZED FORM

For clarity, throughout the remainder of this thesis the following naming convention will be used: N^3 denotes the nonlinearized direct treatment by DVOGER as solved in Ref. 3. "Linearized approximate" indicates the linearized technique based on element level multiplication of C by $\psi_{t-\Delta t}$. The "linearized exact" method, however, is the element level multiplication of C by the trial ψ_t. The compact, or Nxp formulation may be used with any of the linearized methods in addition to the CRANKO treatment discussed later.

A. GENERAL TREATMENT OF THE MATRIX EQUATION

Solution of the matrix formulation of Eq. (8) involves clearing the LHS to provide a discrete relation for each $\dot{\psi}_k$. This is accomplished either by iteration, which yields an approximation, or by matrix inversion, which has the advantage of giving the individual $\dot{\psi}_k$ explicitly. In both the nonlinearized N^3 and the linearized NxN forms, Eq. (8) is reduced with multiplication by $[A]^{-1}$; thus

$$[A]^{-1}[A]\{\dot{\psi}\} = [A]^{-1}[B]\{\psi\} + [A]^{-1}[C\phi]\{\psi\} \tag{20}$$

in the linearized $\psi^2 = \phi\psi$ case or as

$$[A]^{-1}[A]\{\dot{\psi}\} = [A]^{-1}[B]\{\psi\} + [A]^{-1}[C]\{\psi^2\} \tag{21}$$

for the direct, nonlinear, formulation. This results in

$$\{\dot{\psi}\} = [AB]\{\psi\} + [A^{-1}][C^*]\{\psi\} \tag{22}$$

where

$$[C^*] = [C\phi] \quad \text{or} \quad [C]\{\psi\} \tag{23}$$

as appropriate. The point here is that the $[AB]$ and the $[A^{-1}]$ are formed once and for all based on the time independent properties and geometry of the problem, whereas the $[C\phi]$ must be computed for each time t. Although the construction of the $[C]$ is required only once in the direct (N^3) method, it must be additionally multiplied by $\{\psi^2\}$ at each time step.

B. PRACTICAL CONSIDERATIONS WITH THE LINEARIZED TECHNIQUE

Relating Eq. (22) to the demands of the subroutine DVOGER, which involves the evaluation of $\dot{\psi}$ for every trial value of ψ, it is apparent that the construction of $[A]^{-1}[C\psi]\{\psi\}$ for each of these trials could be a time consuming process, and hence a serious drawback of the linearized treatment. Noting, however, that for small $\Delta t, \psi_t \approx \psi_{t-\Delta t}$, the approximation $[C\psi] \approx [C\psi]$, where $\phi = \psi_{t-\Delta t}$, overcomes this disadvantage by allowing the construction of $[A]^{-1}[C\phi]$ only once for a given time t, regardless of how many trial solutions are attempted by DVOGER. Equation (22) can now be further simplified since $[A]^{-1}[C\phi]$ is readily combined with $[AB]$ to reformulate the problem as

$$\{\dot{\psi}\} = [C^{**}]\{\psi\} \ . \tag{24}$$

Thus, although the linearized form can be handled exactly, i.e., with $\phi = \psi$, it may be more expeditous to establish the $[C^{**}]$ only once per valid time point, (that is, where the convergence criterion of the equation solution has been satisfied). In this case, the linearized treatment is then an approximate technique in that $\phi = \psi_{t-\Delta t} \neq \psi_t$.

C. DIFFICULTIES WITH THE APPROXIMATION

The simplification expressed as Eq. (24) works extremely well when used with DVOGER as long as large changes in ψ are incurred by small Δt (the so-called "stiff" system). When the solution approaches steady state, however, $\dot{\psi}$ approaches zero faster than $(\psi_t - \psi_{t-\Delta t})/\Delta t$ does, and this causes a second kind of perturbation wherein the system becomes increasingly sensitive to small changes in $\psi_t - \psi_{t-\Delta t}$. Apparently DVOGER requires very accurate $\dot{\psi}$ information. The $\dot{\psi}(t)$ provided from $\psi_{(t)} \times \psi_{(t-\Delta t)}$ does not match the DVOGER prediction of $\dot{\psi}_{(t)}$, which is based on the assumption that $\dot{\psi}_{(t)}$ was generated by $\psi^2_{(t)}$. This discrepancy in the neighborhood of $\dot{\psi} \approx 0$ causes DVOGER to be unable to recognize the steady state solution. Prepared especially for stiff systems, the equation-solver expects, instead, a large scale change in ψ. The time step is therefore reduced accordingly, which means that very slow progress is made in the steady-state region which is otherwise handled quite rapidly under the exact methods. The implication here is that, in those cases which reach terminal flux values rather early in problem time, the exact formulation of the linearized treatment is to be preferred, even though the $[C^{**}]$ must be computed for each attempted $\psi_{(t)}$. This aspect has been analyzed for the test problems considered, with the results given in Table II.

IV. THE Nxp COMPACTING SCHEME

A. ESTABLISHED COMPACTING METHOD

The problem under consideration here, being of the form $A_{ij}\dot{\psi}_j = B_i(\psi_1 \ldots \psi_N)$, is formally handled by multiplying through by $[A]^{-1}$, discussed earlier. For large matrices, the inversion process, and indeed all matrix operations, become extremely costly in terms of machine time and storage. The fact that the matrices may be banded or symmetric does offer significant economy when the matrix is not inverted. For example, a symmetric NxN matrix may be stored as NxN/2. For matrices resulting from finite element formulation, the numbering schedule determines a band width to be used with conventional compacting schemes. In the finite element case, the difference is compared for each node in the mesh. The largest of these differences plus one equals the minimum band width allowed. For example, node 7 in Figure 1 has a maximum difference of 13 - 2 = 11, whereas node 22 has the largest difference (37 - 10 = 27) for the system. Thus the resultant minimum band width would be 28. As the mesh increases in nodal points, the minimum band width must also increase in response, no matter how adroitly the numbers are assigned. Regardless of how sparce a matrix may be, these schemes all ensure that the "compactness" grows commensurately. In addition, the calculation of an inverse becomes ever more unpleasant, especially since this operation on a sparce matrix generally results in a dense, nonsymmetric inverse.

B. BASIS FOR THE COMPACTING SCHEME

In solving a matrix equation $A_{ij}\dot{\psi}_j = B_i(\psi_1, \ldots, \psi_N)$ it is not necessary to form the inverse, but rather, multiply out the left hand side and rearrange to obtain a set of equations:

$$\dot{\psi}_i = (B_i - A_{ij}\dot{\psi}_j)/A_{ii} \tag{25}$$

where $i,j,k = 1, \ldots N$. This procedure is especially suitable for sparce systems since the number of non-zero A_{ij} entities will be small. Further, if these non-zero A_{ij} can be located and tagged with an identifier, then they may be stored together in a dense array, thereby replacing the standard NxN size matrix by an equivalent array of size Nxp, where p is the maximum number of non-zero entries on any one row.

This type of compacting is especially well suited to matrices resulting from finite element formulation, with p determined by the choice of the finite element and the discretized model. In the case under study, a linear triangular element is used such that, when assembled on the global level, it forms, for the purpose of illustration, a network of interlocking hexagonal polygons, each having an apex located directly over its parent central node. Such a pattern is sketched as Figure 2. Note that the sides are indeed the contributing "neighbor" nodes. Boundary nodes will, of course, be missing any "would be" contributors sought in the boundary exterior. In a regularly drawn system, as in Figure 2, an interior node will have six neighbors plus itself for a total of seven contributors. That is to say, that no matter how large the system of mesh points, a relation describing the effect of the system on any given point would contain a maximum of seven non-zero values.

Geometric considerations or a desire to examine some area of the mesh more closely may result in systems of irregular discretization wherein the maximum number of entries may be greater than seven. This analysis may be extended to consider other finite element shapes in a similar manner.

C. CONSTRUCTION OF THE Nxp ARRAY

To utilize this attribute of finite elements to produce the dense Nxp system, it is necessary only to construct a table of nodal points listing their contributors. This table, or connectivity array, is then used as an index to locate the values of the various quantities associated with a particular node. All the standard matrix operations can be performed on these compacted arrays, but the construction of a matrix inverse has no usefulness since a NxN system is then reconstituted from an Nxp array.

As an example, construction of the connectivity vector associated with node 18 in Figure 1 is

$$[18 \quad 17 \quad 27 \quad 26 \quad 19 \quad 14 \quad 13 \quad 12]$$

and the vector associated with node 6 is

$$[6 \quad 11 \quad 7 \quad 21 \quad 0 \quad 0 \quad 0 \quad 0].$$

The contributors may be entered in any order except that, for computational ease, the central, or "parent" node is the first element. These vectors are collected into the node neighbor connectivity array of size Nxp, or, for Figure 1, 38x8.

To multiply a Nxp array of a_{ij} elements by a vector of b_j elements, a search of the connectivity array is performed on the "i"th row to locate a match for the b_j. When the match is found, the values represented by a_{ij} and b_j are multiplied together resulting in a new C_i vector. If the system is correctly formed and compacted, there always will be a match.

The compacting scheme is bound by the geometry of the problem, and as such, has meaning only as it pertains to that problem. For example, operations with two equi-sized Nxp arrays of different connectivity relations cannot be performed. This is in contrast to regular matrix operations where the restriction is only to size and not to origin. A short computer program to demonstrate the operation of this Nxp scheme is given, with results, in Appendix C.

V. THE CRANK-NICOLSON FORMULATION

Recognizing the very sparce nature of the system of differential equations formulated by this problem and, if the matrix inversion process is to be avoided, the necessity of iteration, a straightforward attack based on the definition of the derivative (finite difference) appears as a feasible alternative to the DVOGER technique. One such approach was presented by J. Crank and P. Nicolson in 1947 [Ref. 7] and has been discussed in many works throughout the ensuing years. This method has been shown to be unconditionally stable. An adaptation of this method is given here as follows:

In the general sense, a matrix formulation of a set of linear differential equations, can be represented as

$$A_{ij}\dot{\psi}_j = C_{ij}\psi_j + F_j(t) \qquad i,j = 1, \ldots, N \qquad (26)$$

where ψ is a function of time. For the case of an initial disturbance as the forcing function, $F(t) = 0$ for $t > 0$. Focusing attention for the moment on a particular equation and expressing [16] in terms of the definition of the derivative

$$A\left[\frac{\psi_t - \psi_{t-\Delta t}}{\Delta t}\right] = C\left[\frac{\psi_t + \psi_{t-\Delta t}}{2}\right] \qquad (27)$$

which is

$$[A - 1/2\cdot\Delta t\cdot C]\psi_t = [A + (1/2\cdot\Delta t\cdot C)]\psi_{t-\Delta t} \qquad (28)$$

$$[(2/\Delta t\cdot A)-C]\psi_t = [(2/\Delta t\cdot A)+C]\psi_{t-\Delta t} \qquad (29)$$

letting $D = (2/\Delta t \cdot A) - C$, $E = (2/\Delta t \cdot A) + C$, $\phi = \psi_{t-\Delta t}$ then, returning to the system form the result is

$$D_{ij}\psi_j = E_{ij}\phi_j = E_i^* \qquad i,j = 1, \ldots, N . \tag{30}$$

This constitutes a set of linear simultaneous equations which may be solved by any convenient method. Chosen here is Gauss-Seidel iteration [Ref. 8] which is well suited for the problem at hand since, the sum of $D_{ij}\psi_i$ and $E_{ij}\phi_j$ will contain very few non-zero terms, thereby incurring relatively small roundoff errors regardless of system size.

This procedure, which also uses the Nxp compacting, has been written for this thesis as the computer code CRANKO. After receiving the matrix information and control parameters from the calling program, CRANKO first builds the C matrix of Eq. (26) for the initial ϕ value. Then it selects a trial time interval and, forming the equation set (30) based on this Δt, attempts to find a solution satisfying a specified convergence criterion based on a relative error. If no satisfactory solution set is found after a number of iterations, a smaller Δt is selected; and, after recomputation of Eq. (30), another attempt at convergence is made. The successful solutions then replace the previous ψ thereby forming the starting point for another cycle, beginning with a new computation of the C matrix. The choice of Δt is the controlling factor. The scheme employed counts the number of iterations required for a solution. When an increasing number of iterations indicate greater difficulty, the interval Δt is decreased; whereas, if fewer iterations are required, the time step may be relaxed and made larger. Experience with the CRANKO routine establishes this method as an extremely fast technique for solving this type of problem.

For those readers unfamiliar with the Gauss-Seidel iteration scheme, a simple example using a 3x3 system is given:

$$D_{ij}\psi_i = E_j^* \qquad \phi = \psi_{t-\Delta t}$$

$$\begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

first iteration:

$$d_{11}\psi_1 + d_{12}\phi_2 + d_{13}\phi_3 = e_1$$

$$\psi_1 = [e_1 - (d_{12}\phi_2 + d_{13}\phi_3)]/d_{11}$$

$$\psi_1^* = \psi_1$$

$$d_{21}\psi_1^* + d_{22}\psi_2 + d_{23}\phi_3 = e_2$$

$$\psi_2 = [e_2 - (d_{21}\psi_1^* + d_{23}\phi_3)]/d_{22}$$

$$\psi_2^* = \psi_2$$

$$d_{31}\psi_1^* + d_{32}\psi_2^* + d_{33}\psi_3 = e_3$$

$$\psi_3 = [e_3 - (d_{31}\psi_1^* + d_{32}\psi_2^*)]/d_{33}$$

for successive iterations,

$$d_{11}\psi_1 + d_{12}\psi_2^* + d_{13}\psi_3^* = e_1$$

$$\psi_1 = [e_1 - (d_{12}\psi_2^* + d_{13}\psi_3^*)]/d_{11}$$

$$\psi_1^{**} = \psi_1$$

$$d_{21}\psi_1^{**} + d_{22}\psi_2 + d_{13}\psi_3^* = e_2$$

$$\psi_2 = [e_2 - (d_{21}\psi_1^{**} + d_{13}\psi_3^*)]/d_{22}$$

$$\psi_2^{**} = \psi_2$$

$$d_{31}\psi_1^{**} + d_{32}\psi_2^{**} + d_{33}\psi_3 = e_3$$

$$\psi_3 = [e_3 - (d_{31}\psi_1^{**} + d_{32}\psi_2^{**})]/d_{33}$$

etc.

VI. SUMMARY OF AVAILABLE METHODS

For clarity, the innovative computational tools introduced in this thesis are summarized. First is the linearizing of the governing equation Eq. (8) by the multiplication on the element level of C_{ijk} by ψ_j. From this development, two options appear feasible: either the exact computation of $[C^*]$ for each trial ψ, or the approximation of $\psi_t \approx \psi_{t-\Delta t}$ to allow the construction of $[C^*]$ only once during the search for a new ψ_t. This approach uses, as does the "direct," or nonlinear treatment involving $[C]$, the inverting of $[A]$ to clear the LHS as shown in Eq. (22).

The second innovation is the reduction of the NxN matrix to a smaller Nxp compact form based, not upon the mathematics of the problem, but upon the geometry of the finite element model. The same solution methods may be used, with the only difference being that the inverting of $[A]$ is replaced by iteration. This iteration can only be as accurate as the imposed error criterion. For large systems, however, where the $[A]^{-1}$ produces a full NxN matrix on the RHS, this iteration is especially beneficial.

The last contribution was the development of a new equation-solver based on a different theory than the DVOGER routine. The new code deals with the differential equations directly and employs the Nxp compacting.

VII. TEST PROBLEMS AND RESULTS

A. THE PHYSICAL MODEL

A cylindrical nuclear reactor consisting of a core region and blanket having overall a height of 220 cm and a diameter of 180 cm was modeled for the test problems. As shown in Figure 1, a radial slice from this reactor was discretized by a finite element of 54 elements and 38 nodal points. After a detailed analysis of this mesh was completed, preliminary investigation was begun using a model of 220 elements (Figure 3).

B. COMPUTER PROCESSING CONSIDERATIONS

All programs were written in the FORTRAN IV language and processed on the IBM 360/67 computer using the FORTRAN 'H' compiler. The results presented in terms of CPU storage requirements and processing time ought to be indicative, in a relative way, of those obtained from other machines. Single precision (seven significant digits) was used throughout the investigation, with the results later verified using double precision (fifteen significant digits). The observed relative discrepancy was less than .01%. It is recongized, however, that for larger systems a more significant difference may result.

C. PROBLEM ANALYSIS

The three techniques discussed have been incorporated into computer codes and have been tested on those problems described in Reference 3, namely, the dynamic reactor response to 1) a disturbance at the center, 2) a uniform disturbance throughout the core, and 3) a disturbance occurring at a skew point in the core ($R = 40$ cm, $Z = 0$ cm). The direct, or N^3

matrix (an $Nx(N^2)$ array) equation was solved in Reference 3, and those results are used in this thesis as a standard for comparison. The computer codes developed here have been constructed within the framework and parameters of the original N^3 program, thereby allowing any discrepancy to be attributed to the technique employed. For each technique and disturbance, the flux at three sample points was recorded throughout the time history of the solutions and written onto a computer storage device. A separate program was written to process this information and produce the correlations in tabular and graphic form. The graphic results are presented in Figures 4 - 21 for the center point (R = 0 cm, Z = 0 cm), a core point (R = 40 cm, Z = 40 cm), and a reflector point (R = 75 cm, Z = 80 cm). Table II lists the specific programs employed arranged by disturbance type and gives parameters such as storage requirements and computer processing time.

1. Solutions with the CRANKO Subroutine

All techniques appear to give acceptable accuracy, however there is a startling difference in processing time. The CRANKO subroutine was able to accomplish in seconds the work which required minutes for the DVOGER subroutine (See Table II). The steady state solution given by CRANKO was in every case higher than that obtained by DVOGER. The implication here is not clear since there is no analytical solution available to represent the exact answer. In general, the transient solutions agreed well, except for the oscillation of the CRANKO solution in the early stages, most clearly depicted in the extreme case of Figures 8 and 9. This fluctuation is most likely the result of inadequate control of the CRANKO time step selection where apparently an interval has been chosen which is too large to maintain good accuracy. It is noteworthy,

however, that the technique is so stable that even after large deviation it is able to recognize the error, correct itself, and return to the solution curve given by the DVOGER routine.

2. Solutions with the DVOGER Subroutine

The linearized technique using premultiplication by the previous time step flux ($\psi_{t-\Delta t}$) conformed to theoretical expectations. As the steady state solution was approached, this method proceeded with increasing difficulty. Hence much more processing time to advance through the steady state was required.

The direct N^3 method, on the other hand, has no difficulty in recognizing the steady state and marches forward with large time increments until problem termination. A very successful effort was made to use the exact treatment with the linearized technique. As expected, more processing time was consumed in getting to the steady state due to the requirement to recompute the $[C^* \psi]$ for each trial ψ, but once the steady state was approached, the procedure progressed rapidly as in the direct treatment. Comparing the direct with the linearized exact method, theory developed in this thesis predicts identical results with twice the computing time required by the linearization. As shown in Table II, this is indeed the case. The largest relative error observed between the two methods was .06%, which is attributed to roundoff steming from the doubling of the computational requirements. When the greatest accuracy is required, the exact treatment with linearization may therefore prove the most useful, especially if the problem rapidly reaches steady state. This discussion of the linearized technique is applicable both to the NxN method or to the Nxp compact method. However since the Nxp method requires iteration, more processing time and less accuracy might be expected.

D. EVALUATION OF ERROR

The error analysis for these procedures must remain subjective as there is no "correct" result. Several sources of error are present and deserve comment. Tests were made with both EPS = .1 and EPS = .01, the error criterion for the DVOGER routine, and little difference was noted for the large increase in processing time. [EPS is defined by DVOGER as $\sum_{i=1}^{N} \frac{\text{ERROR (i)}}{Y_{\text{MAX}}}$]. The convergence criterion [relative deviation of any ψ_i between successive iterations] for the iterative solutions in CRANKO was varied by tenths from .01 to .001 with the result of producing higher steady state values of only a few percent as the criterion was made more stringent. Processing time for this variation increased only a few seconds. Double percision trials for the linearized DVOGER method and the compact CRANKO were made with neglibible difference.

The grid of only 54 elements theoretically introduces the largest source of error from the "true" solution. With the techniques developed in this thesis, a finer grid size of 220 elements has been implemented into the CRANKO and the linearized codes. Because the linearized code (DVOGER) requires about 566K core size, extensive testing with this technique is not anticipated, but the CRANKO routine which requires only 188K is currently being investigated. As the results of this investigation become clear, a better understanding of the error induced by each of the various approaches to this problem would be obtained.

VIII. CONCLUSION

It appears that nonlinear differential equations resulting from a finite element formulation (or for that matter finite differencing) may be succesfully reduced using the linearizing technique developed in this thesis. The compacting scheme for matrix equations effects gross savings for storage of large systems (Table I) and becomes even more attractive when the inverse of a matrix is either not desired or required. The Crank-Nicolson formulation coupled with Gauss-Seidel iteration as expressed in the computer code CRANKO has demonstrated itself to be a viable, extremely fast technique for the solution of differential equations. Further improvements to the CRANKO routine, specifically in regard to better control of the time step selection, may improve upon the attractiveness of this approach.

Development is warranted in several areas. The computation of the Jacobian using iteration must be formulated in order to use the DVOGER subroutine with the compact system. DVOGER itself might usefully be reviewed to reduce its size requirements also. Investigation of the problems using the fine mesh size of 220 elements is continuing. This should yield more exact results and offer a better standard for comparison.

APPENDIX A

TABLES

TABLE I

IBM 360/67 CORE REQUIREMENTS
in K Bytes

SYSTEM	NO. NODES	OVERHEAD	MATRIX STORAGE	TOTAL
NxN^2	38	100	225	325
NxN	38	100	35	135
Nxp	38	100	12	112
NxN	132	150	418	568
Nxp	132	150	38	188
Nxp	400	290	112	402 †

†calculated estimate

TABLE II

COMPARISON OF METHODS
38 NODAL PT. MODEL

METHOD	CORE (k)	PROCESSING TIME (MIN) CENTER	UNIFORM	SKEW	STEADY-STATE ERROR (%)
DVOGER N^3 .01	330	>	5.33	6.27	-
DVOGER N^3 .1	330	5.32	3.28	4.57	<.01
DVOGER NxN exact	130	10.22	6.07	8.10	.06
DVOGER NxN approx	130	10.73	>15.	>10.	.16
CRANKO	110	.62	.20	.22	5.
CRANKO DP	158	.77	.23	.25	5.

Notes:

1. Processing time is that time required by the program to reach one second in problem time. Steady state was obtained for each run.

2. .01 & .1 pertain to DVOGER error criterion EPS. All NxN trials used EPS = .1 CRANKO used iteration convergence tolerance of .0001.

3. Double precision (DP) trials showed a difference in the fifth significant digit when compared to the SP counterparts.

4. NxN approximate technique proceeds as rapidly as the N^3 method until steady state is approached. This has been verified by experiments using various processing time cutoff points.

APPENDIX B

FIGURES

REACTOR MODEL WITH 38 NODES

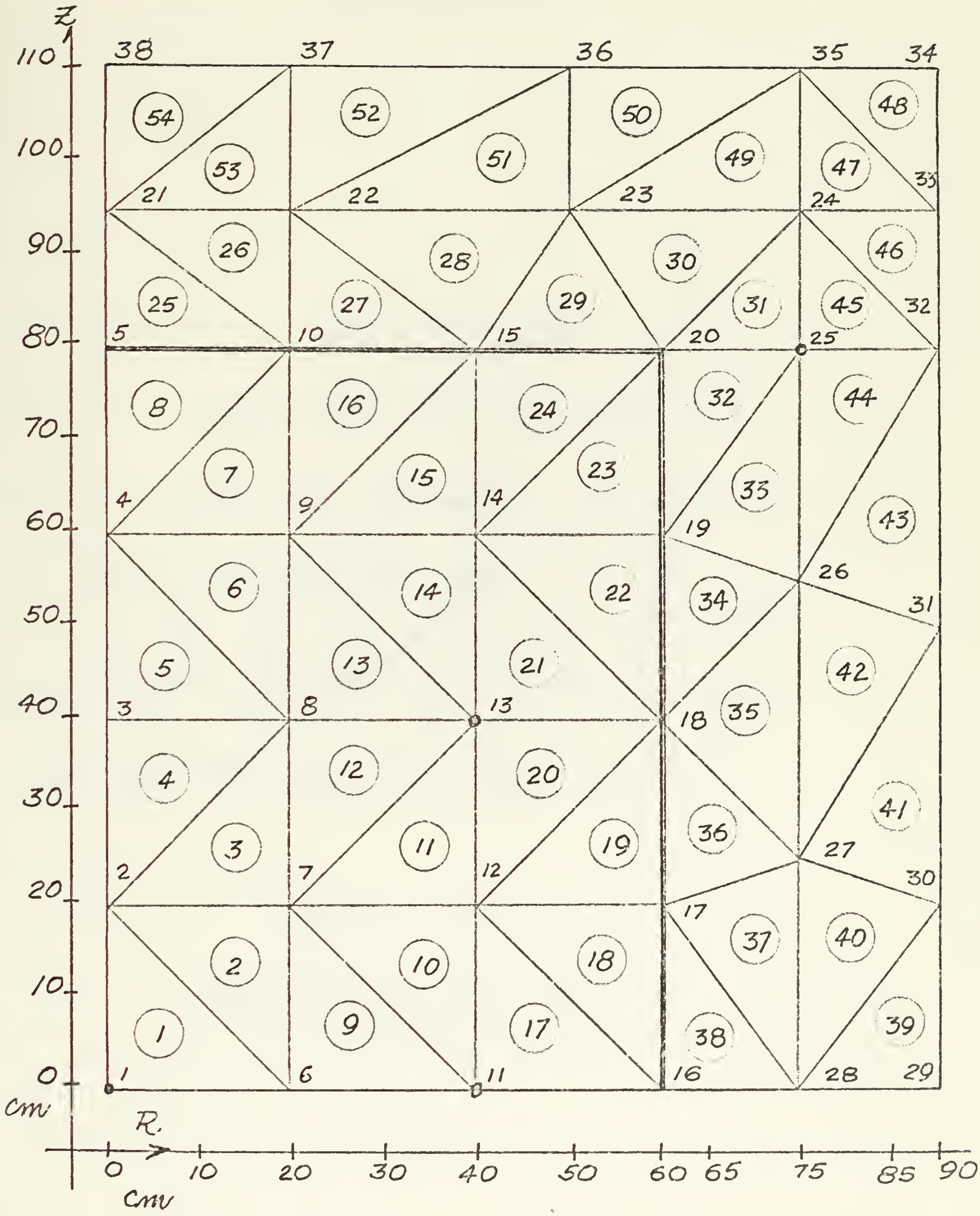
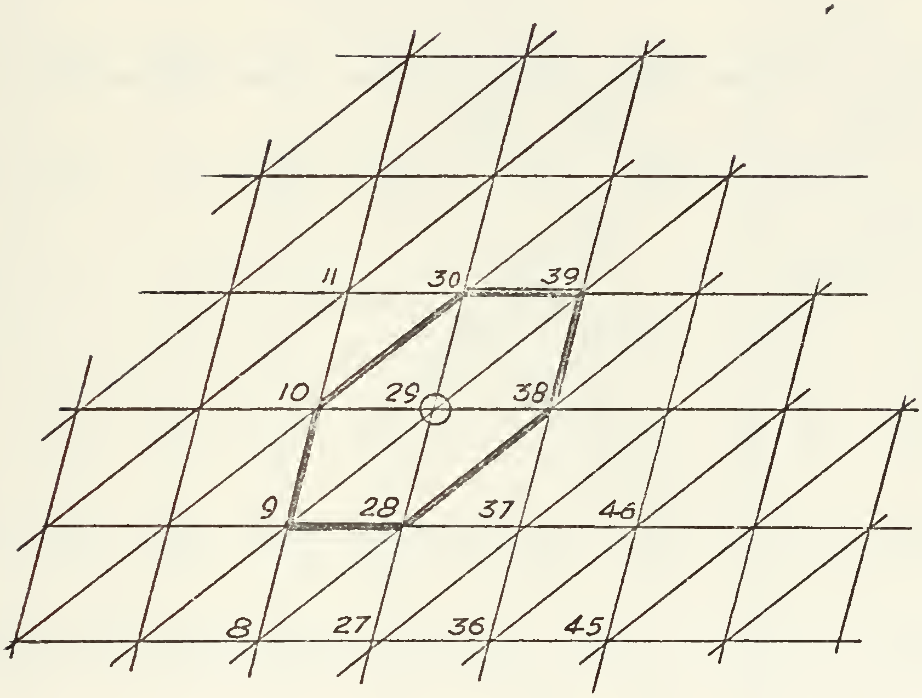


FIGURE 1

NODAL CONNECTIVITY



CONNECTIVITY FOR NODE 29 IS

29 9 10 30 39 38 28

THESE NODES FORM CORNER POINTS OF ALL THE TRIANGULAR ELEMENTS WHICH CONTRIBUTE TO OR INFLUENCE POINT 29

FIGURE 2

REACTOR MODEL WITH 132 NODES

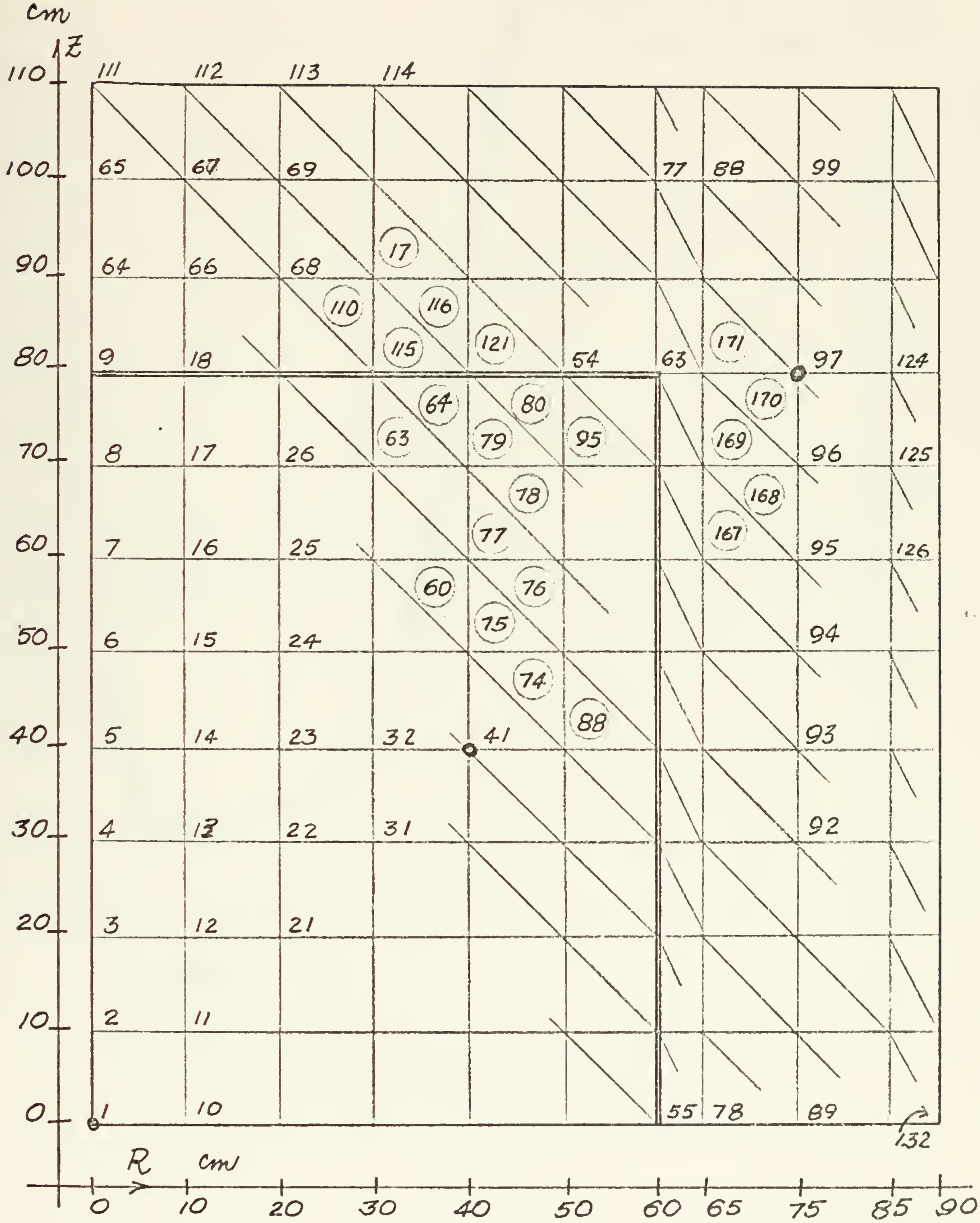
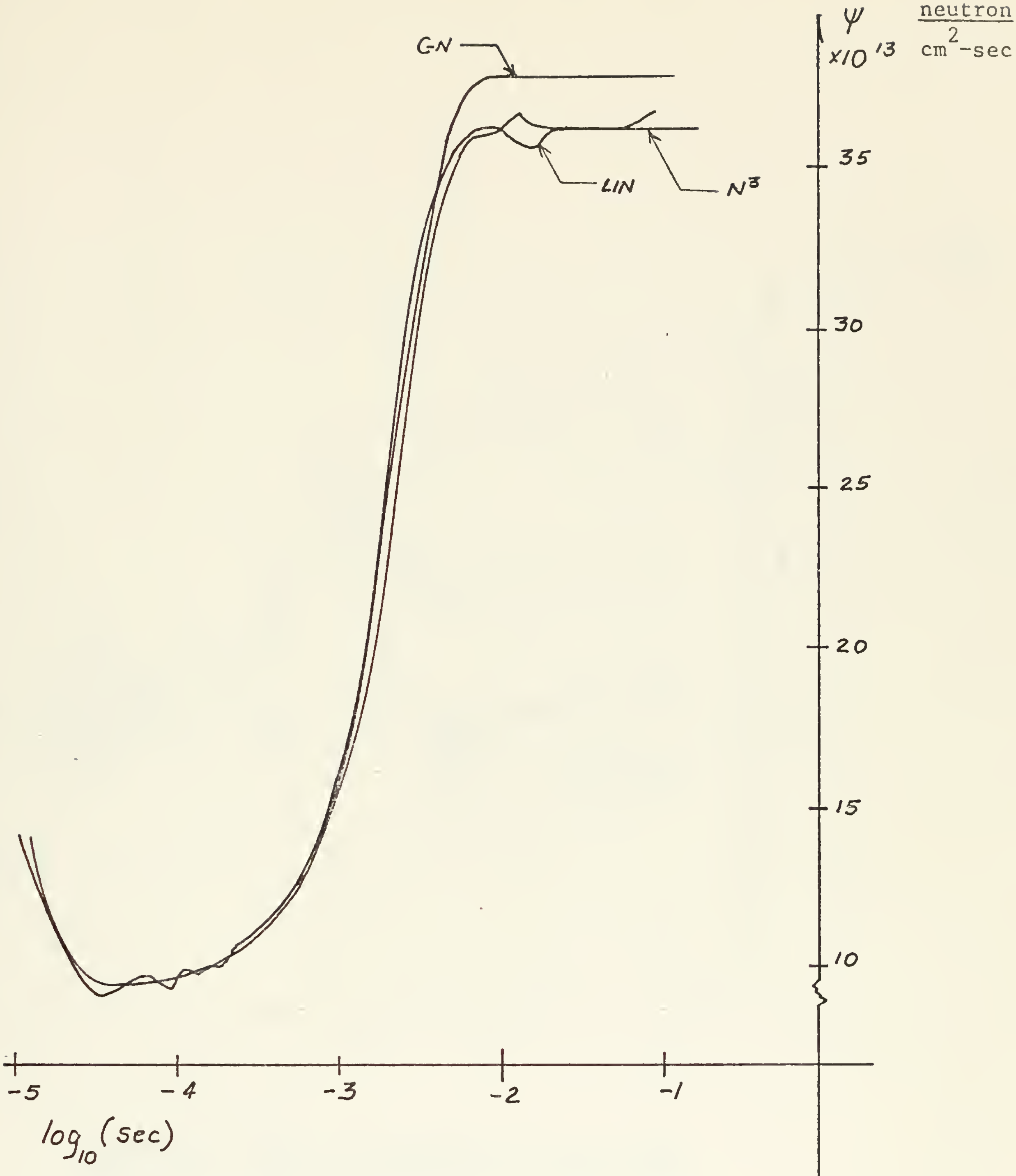
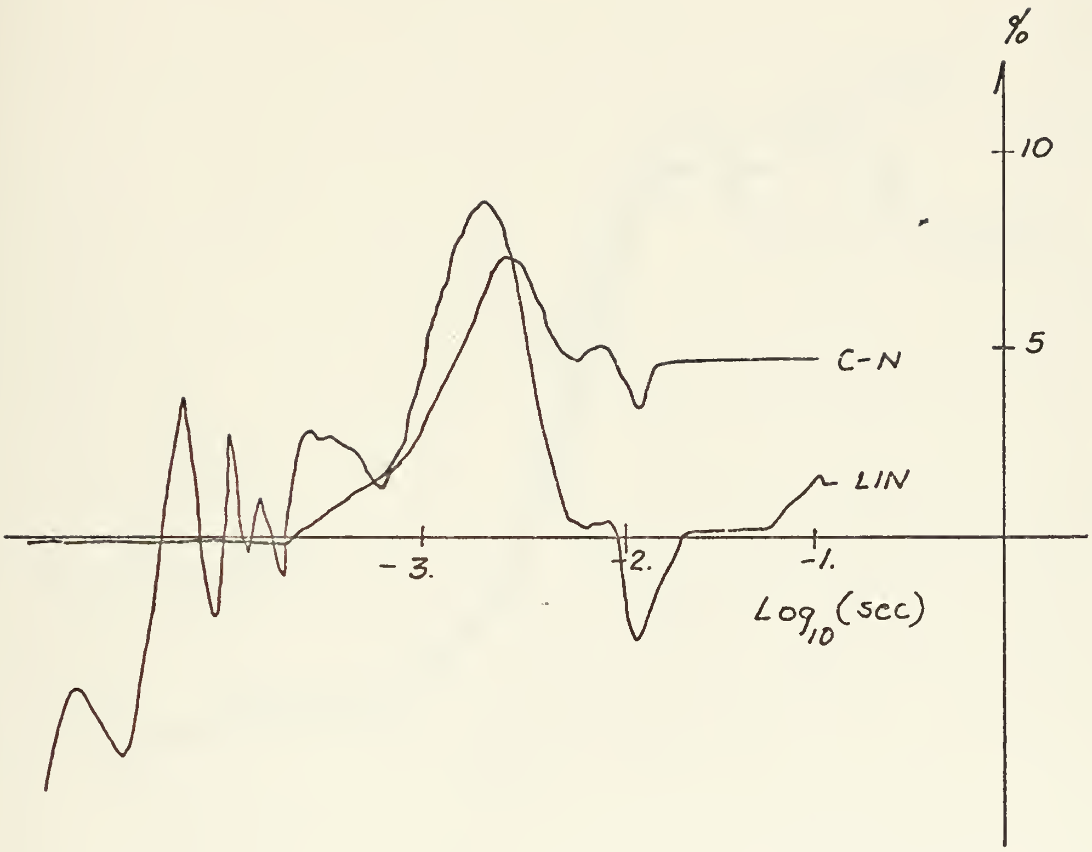


FIGURE 3



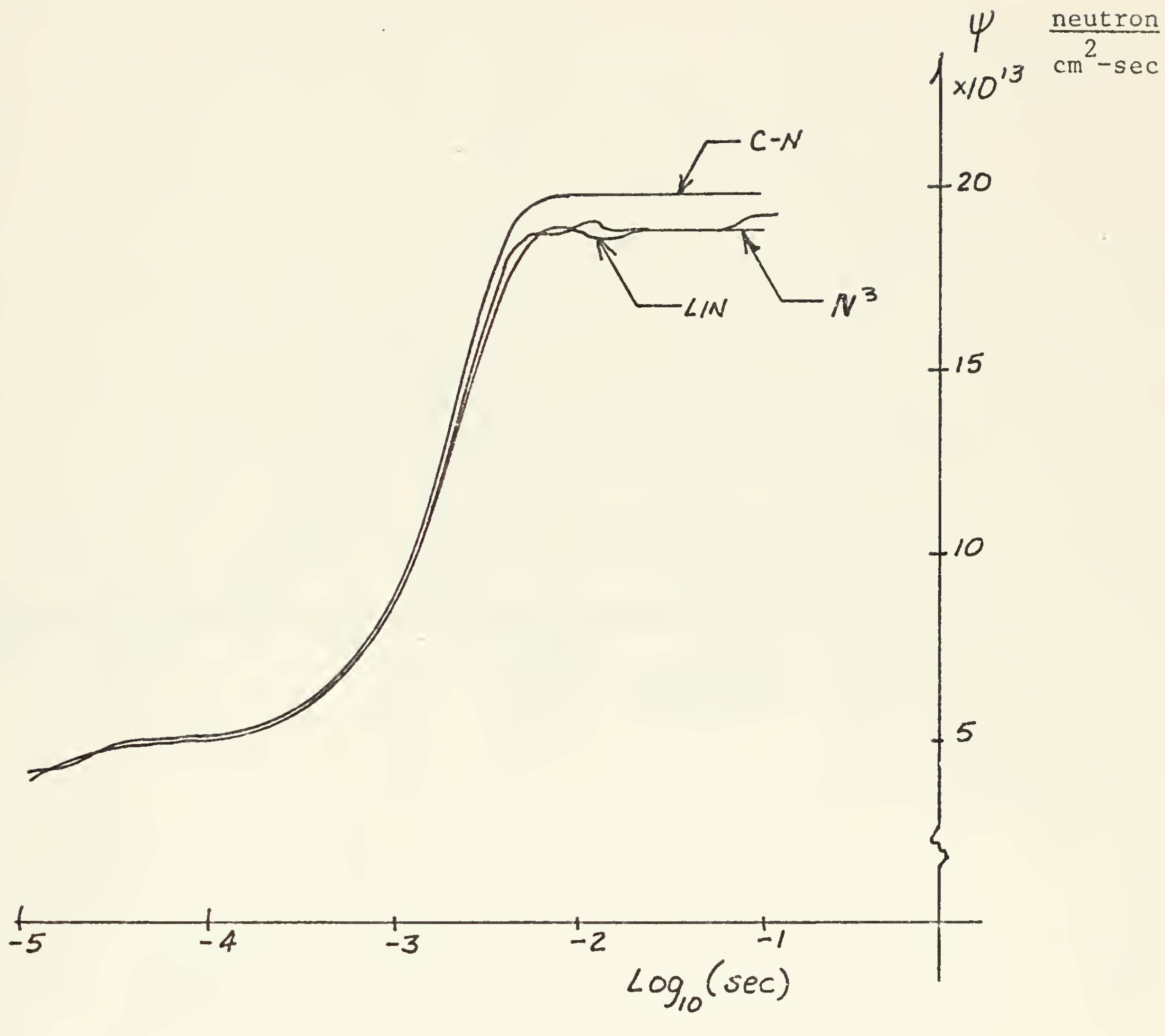
TIME DEPENDENT NEUTRON FLUX AT REACTOR CENTER
FOR A CENTRAL DISTURBANCE
PLOT OF CENTER DISTURBANCE - CENTER POINT

FIGURE 4



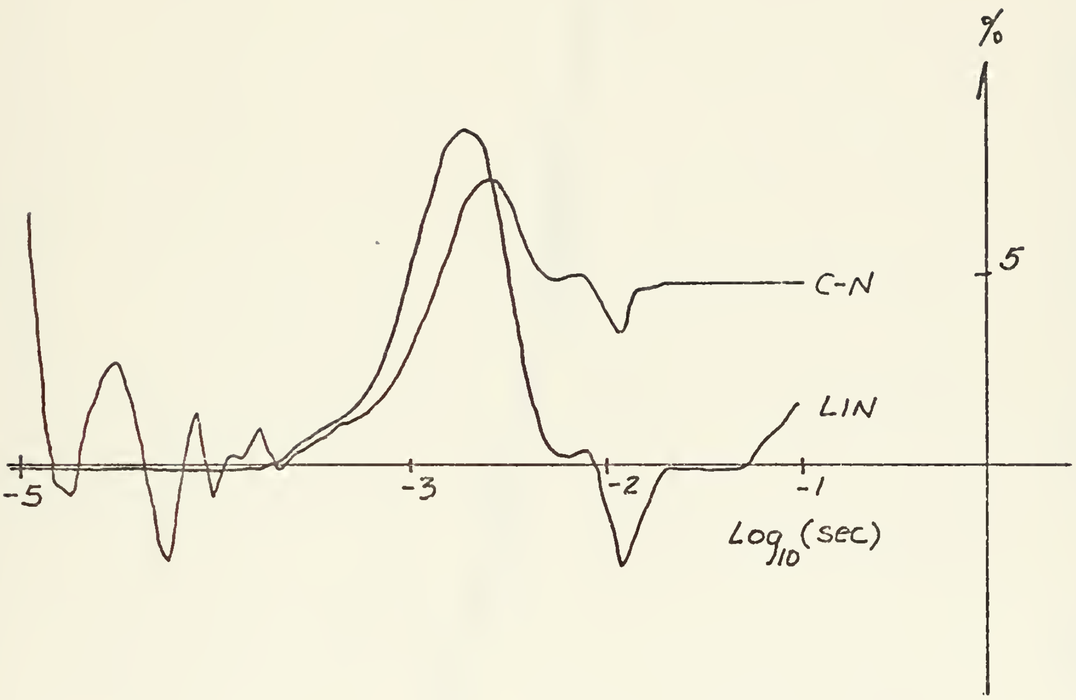
DEVIATION FROM N^3 SOLUTION AT REACTOR CENTER
COMPARISON - CENTER DISTURBANCE - CENTER POINT

FIGURE 5



TIME DEPENDENT NEUTRON FLUX AT CORE POINT Z=40cm, R=40cm
FOR A CENTRAL DISTURBANCE
PLOT OF CENTER DISTURBANCE - CORE POINT

FIGURE 6



DEVIATION FROM N^3 SOLUTION AT CORE POINT Z=40cm, R=40cm
COMPARISON - CENTER DISTURBANCE - CORE POINT

FIGURE 7

PLOT OF CENTER DISTURBANCE - REFLECTOR POINT

TIME DEPENDENT NEUTRON FLUX AT REFLECTOR POINT Z=80cm, R=75cm FOR A CENTRAL DISTURBANCE

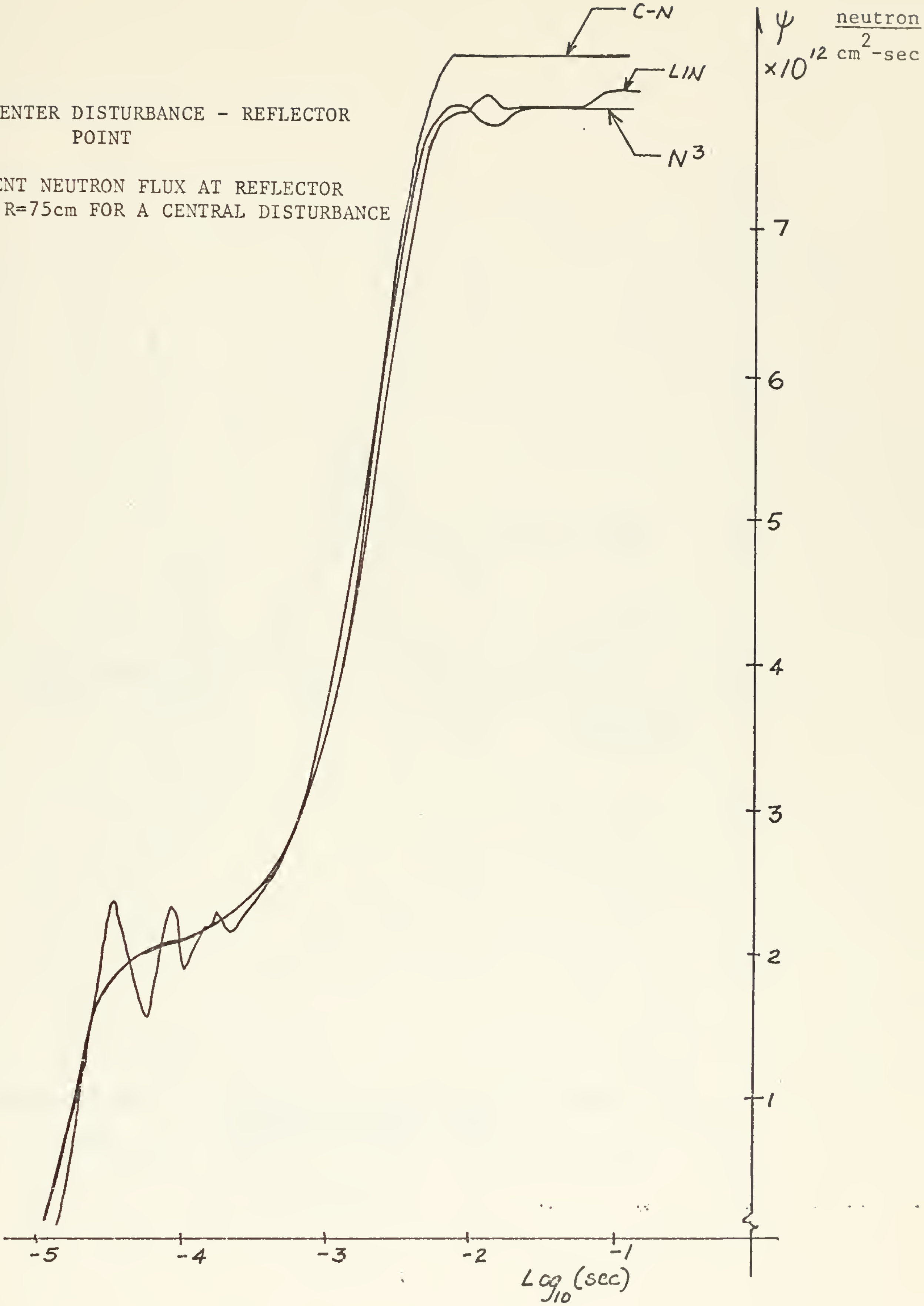
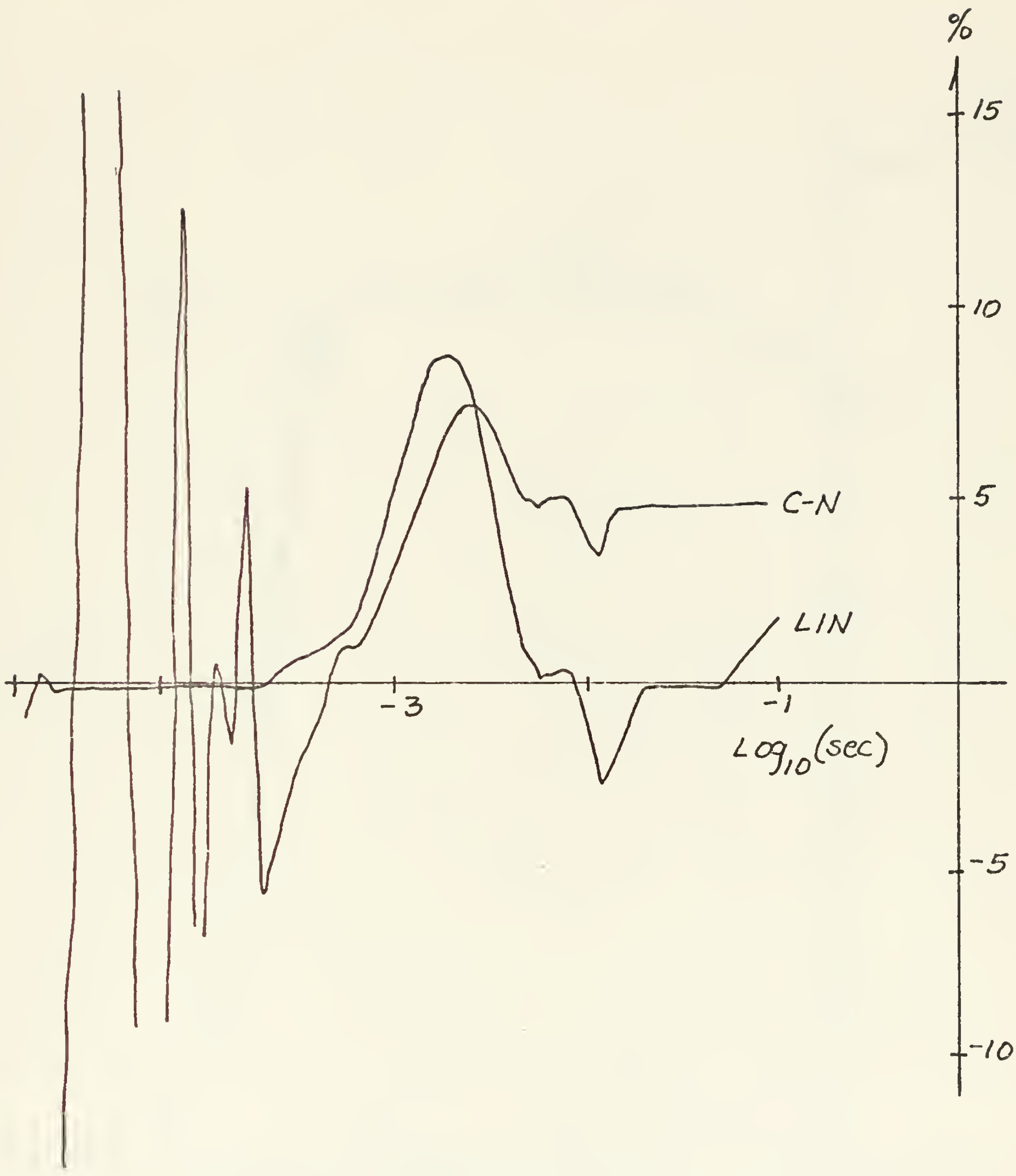
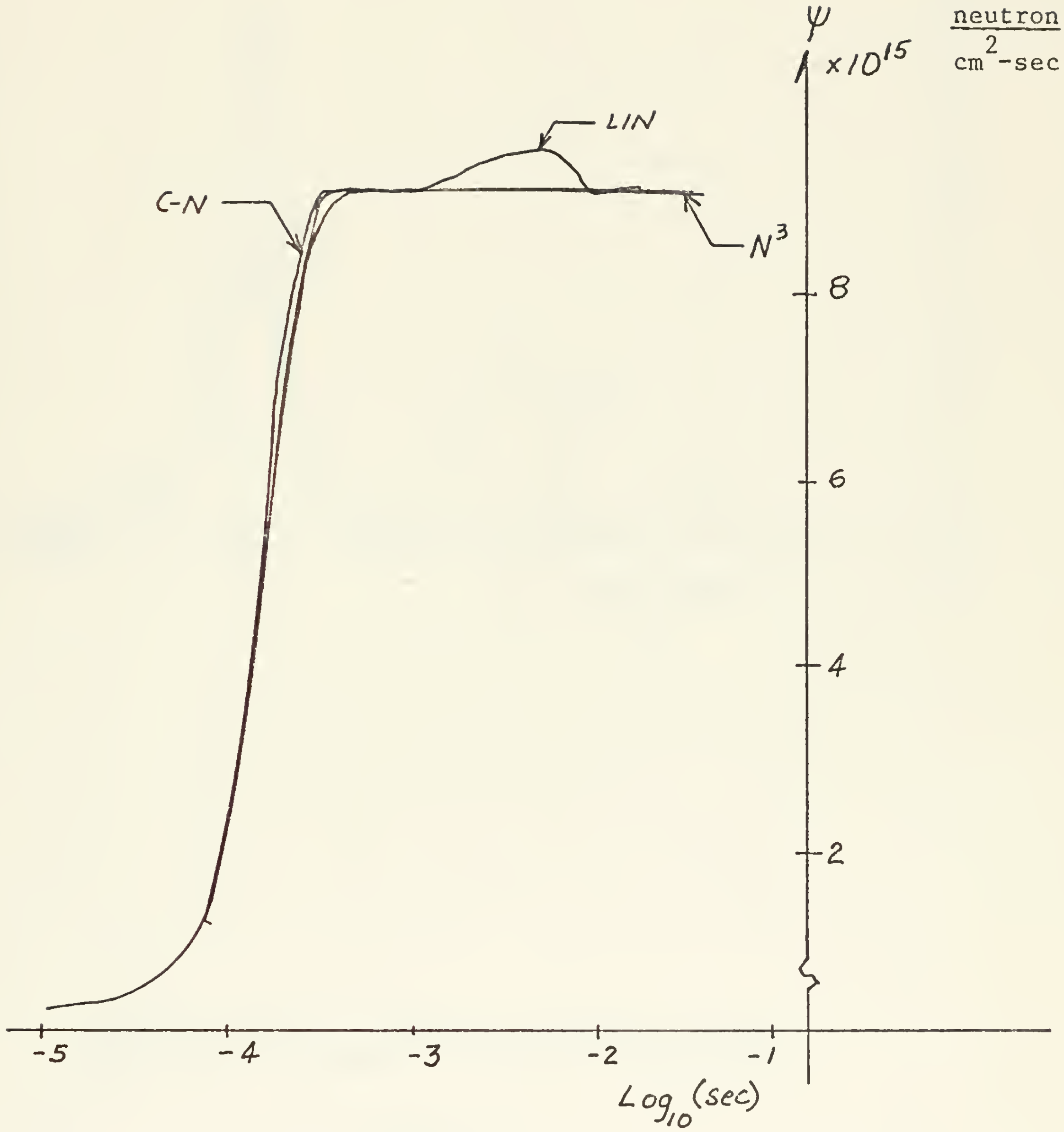


FIGURE 8



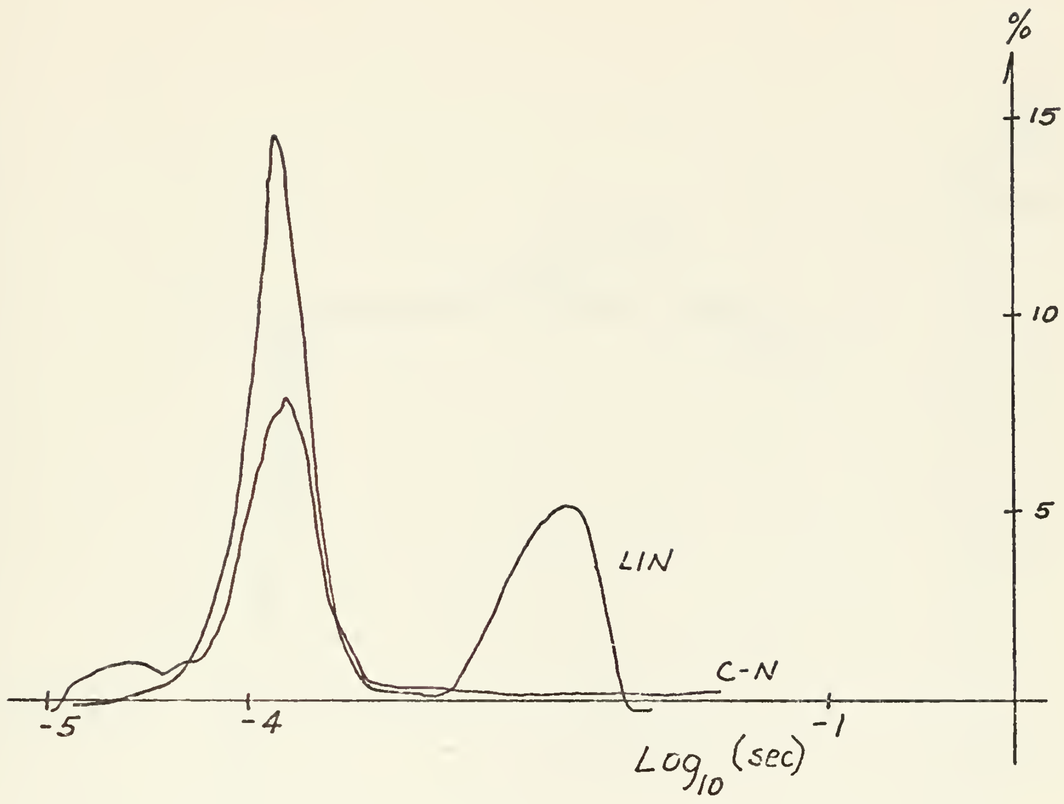
DEVIATION FROM N^3 SOLUTION AT REFLECTOR POINT Z=80cm, R=75cm
AT REFLECTOR TEST POINT
COMPARISON - CENTER DISTURBANCE - REFLECTOR POINT

FIGURE 9



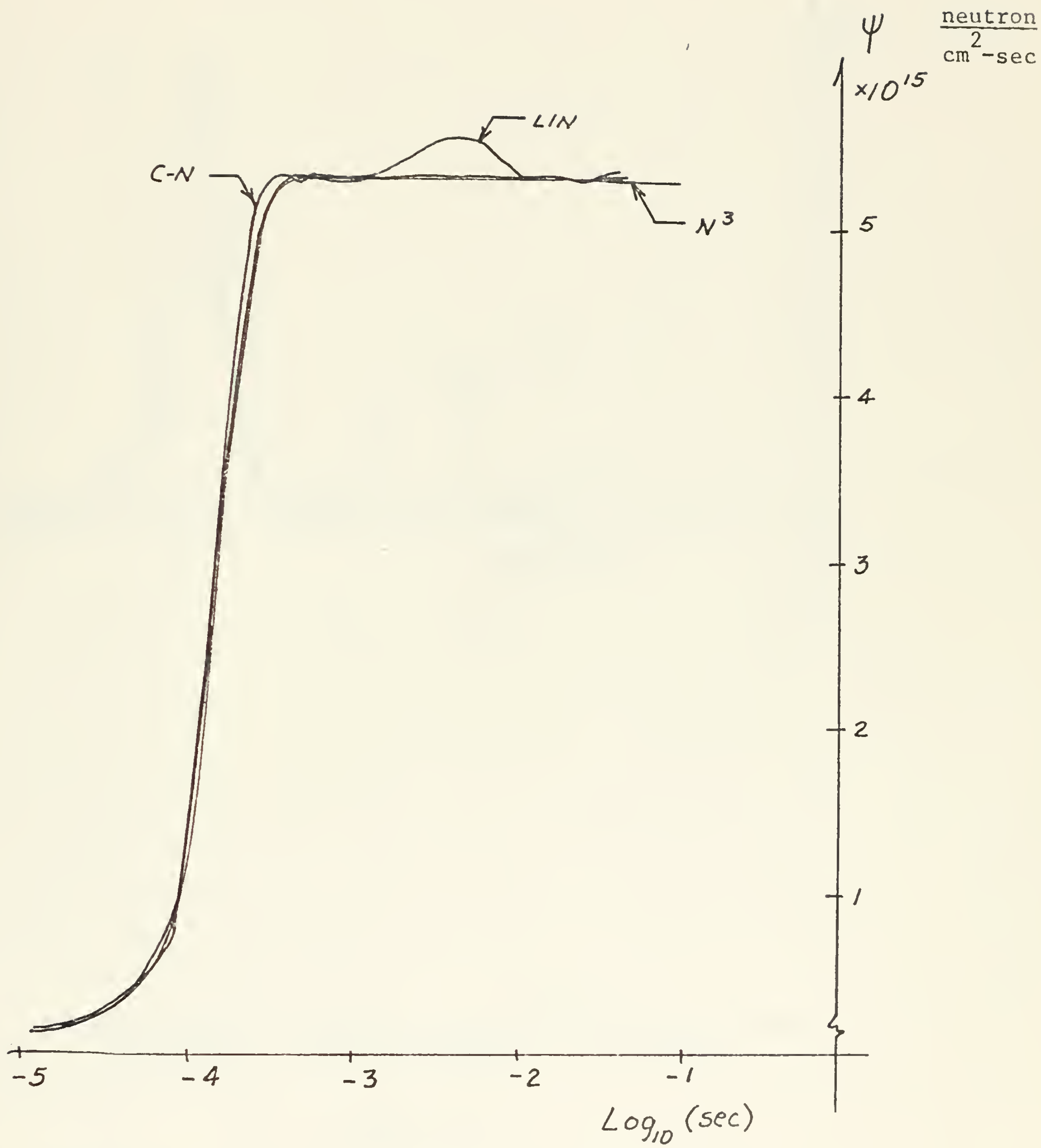
TIME DEPENDENT NEUTRON FLUX AT REACTOR CENTER
FOR A UNIFORM DISTURBANCE
PLOT OF UNIFORM DISTURBANCE - CENTER POINT

FIGURE 10



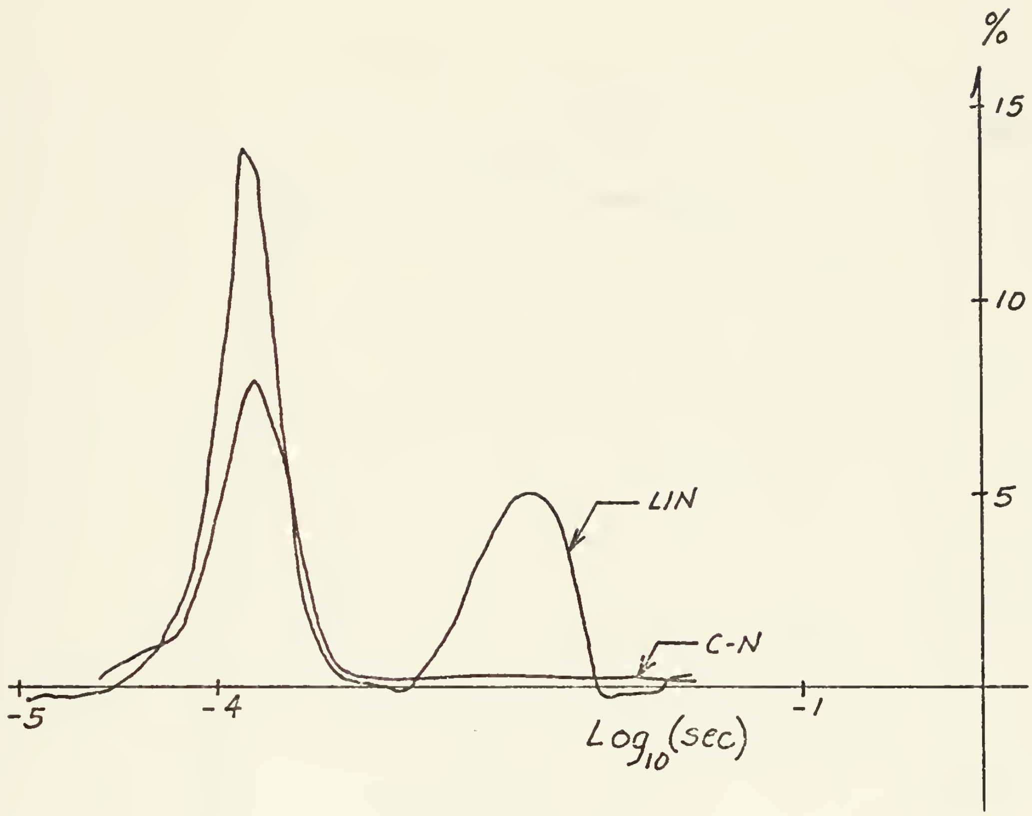
DEVIATION FROM N^3 SOLUTION AT REACTOR CENTER
COMPARISON - UNIFORM DISTURBANCE - CENTER POINT

FIGURE 11



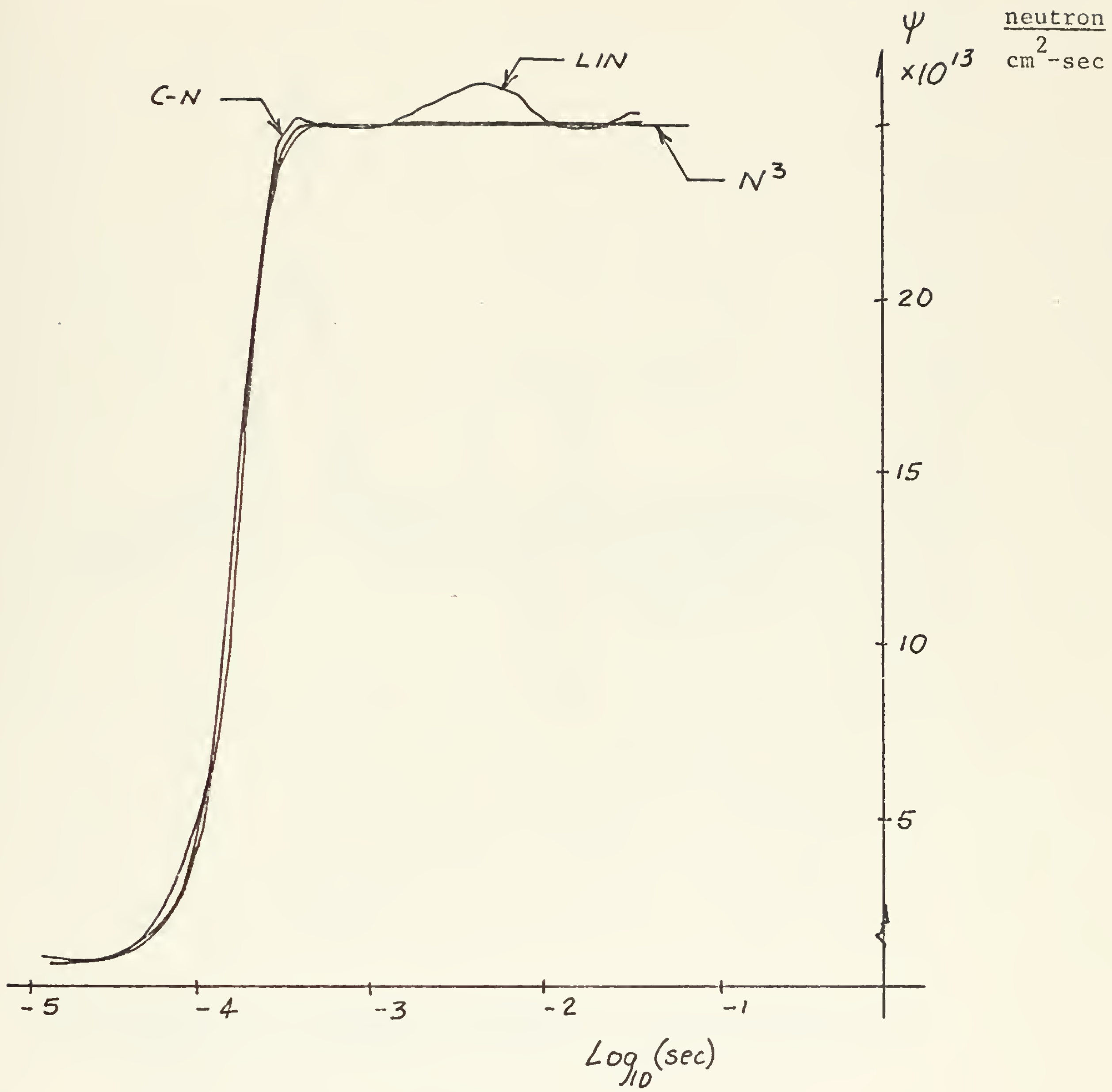
TIME DEPENDENT NEUTRON FLUX AT CORE POINT Z=40cm, R=40cm
FOR A UNIFORM DISTURBANCE
PLOT OF UNIFORM DISTURBANCE - CORE POINT

FIGURE 12



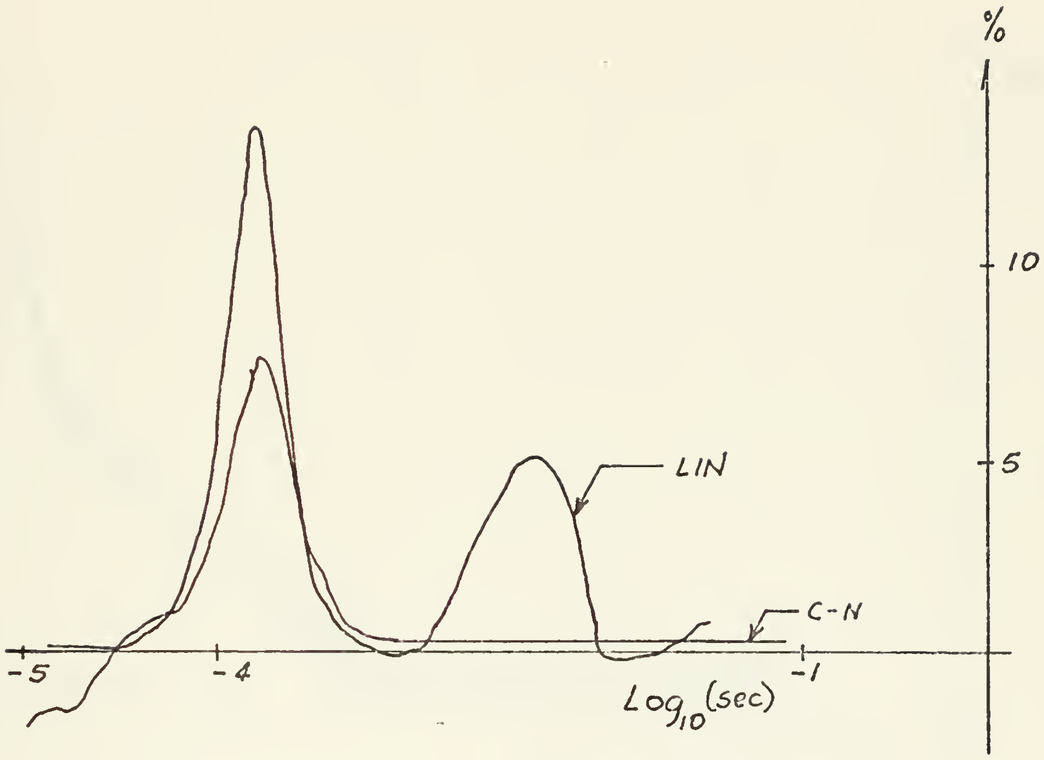
DEVIATION FROM N^3 SOLUTION AT CORE POINT Z=40cm, R=40cm
COMPARISON - UNIFORM DISTURBANCE - CORE POINT

FIGURE 13



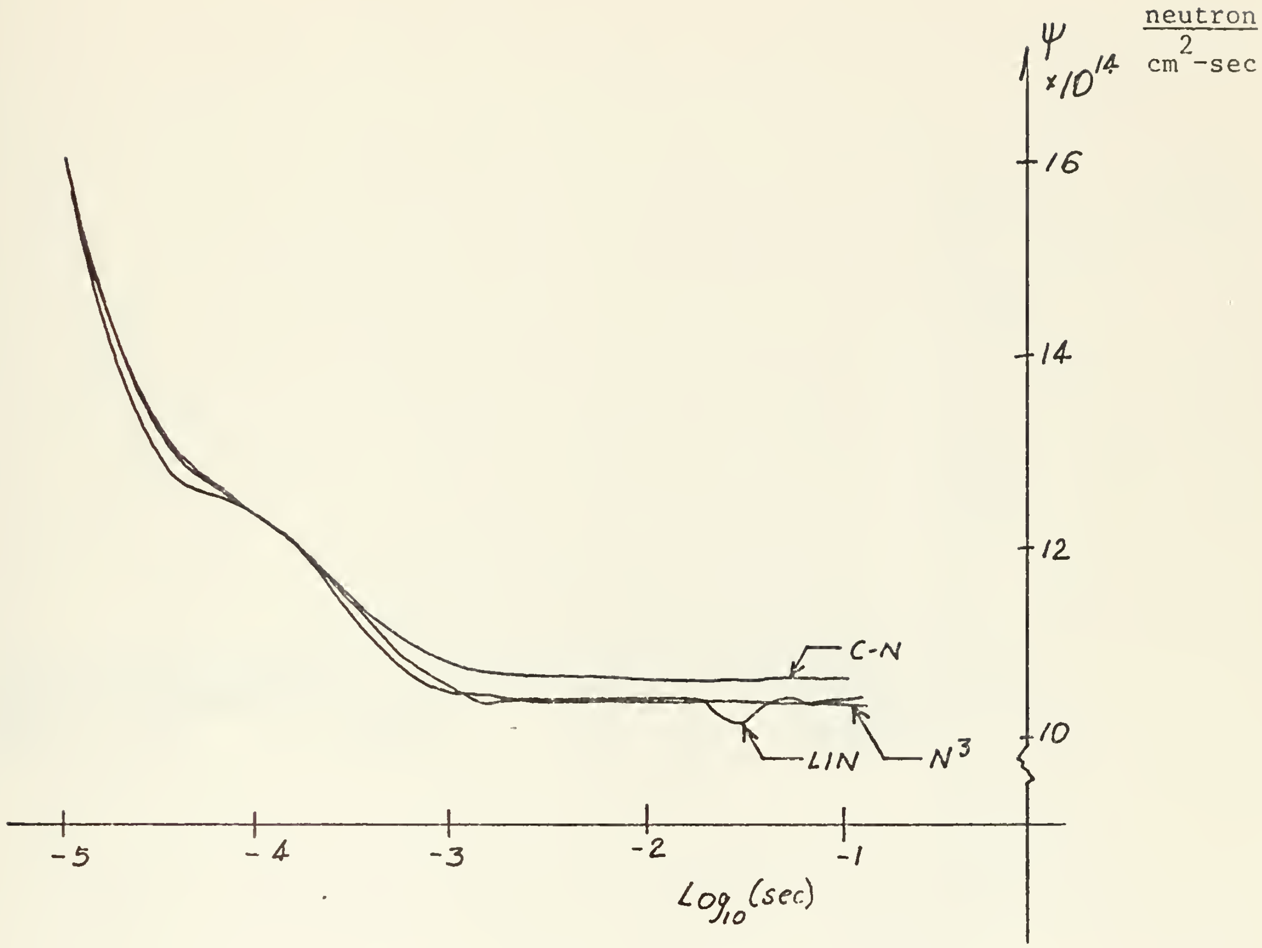
TIME DEPENDENT NEUTRON FLUX AT REFLECTOR POINT Z=80cm, R=75cm
FOR A UNIFORM DISTURBANCE
PLOT OF UNIFORM DISTURBANCE - REFLECTOR POINT

FIGURE 14



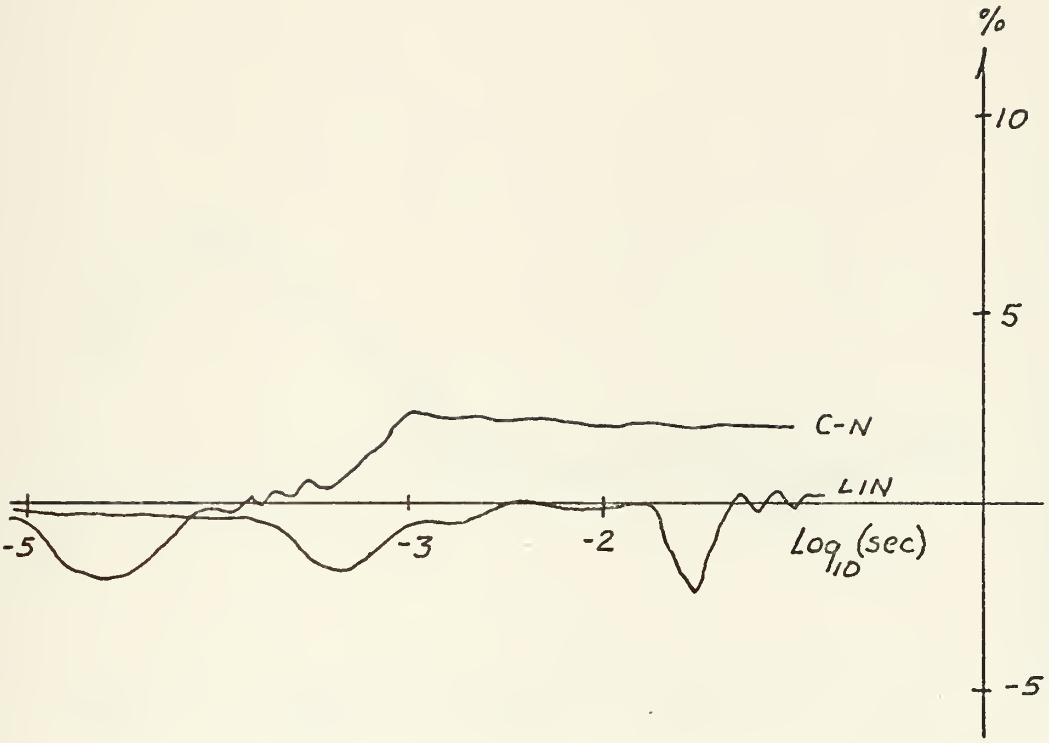
DEVIATION FROM N^3 SOLUTION AT REFLECTOR POINT Z=80cm, R=75cm
COMPARISON - UNIFORM DISTURBANCE - REFLECTOR POINT

FIGURE 15



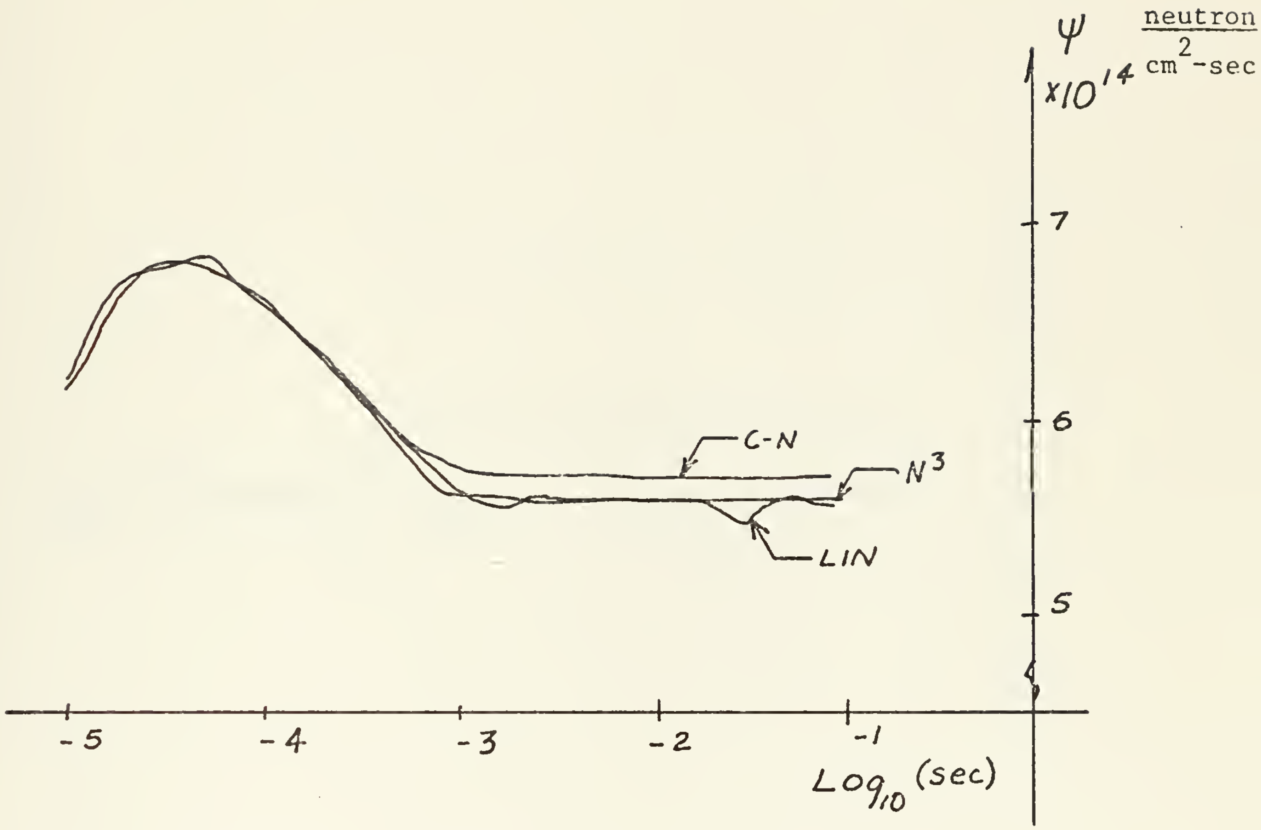
TIME DEPENDENT NEUTRON FLUX AT REACTOR CENTER
FOR A SKEW DISTURBANCE
PLOT OF SKEW DISTURBANCE - CENTER POINT

FIGURE 16



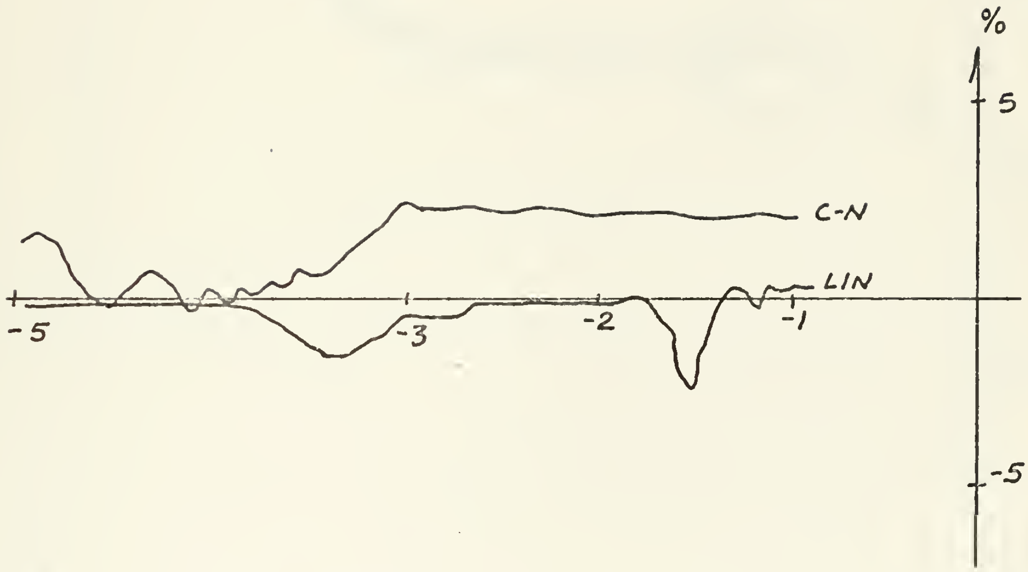
DEVIATION FROM N^3 SOLUTION AT REACTOR CENTER
COMPARISON - SKEW DISTURBANCE - CENTER POINT

FIGURE 17



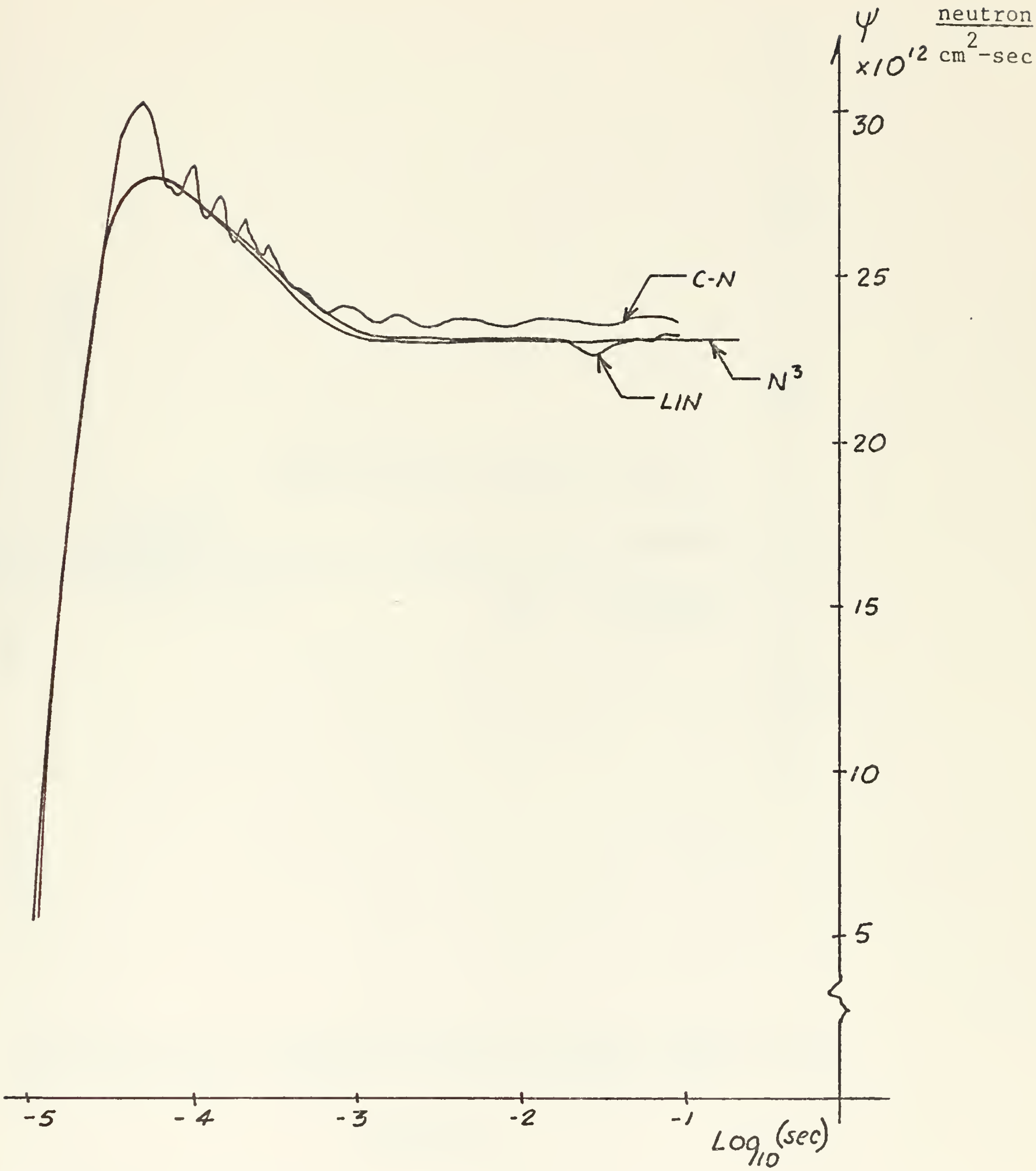
TIME DEPENDENT NEUTRON FLUX AT CORE POINT Z=40cm, R=40cm
FOR A UNIFORM DISTURBANCE
PLOT OF SKEW DISTURBANCE - CORE POINT

FIGURE 18



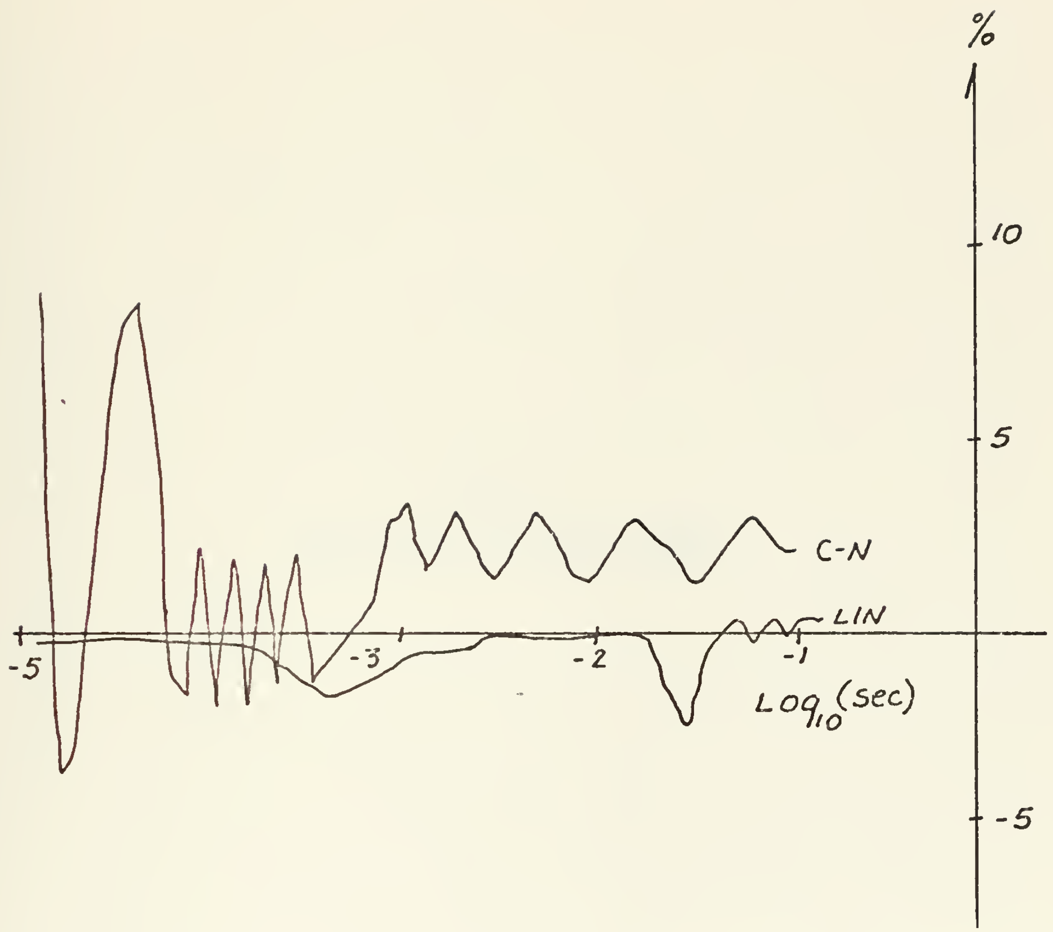
DEVIATION FROM N^3 SOLUTION AT CORE POINT Z=40cm, R=40cm
COMPARISON - SKEW DISTURBANCE - CORE POINT

FIGURE 19



TIME DEPENDENT NEUTRON FLUX AT REFLECTOR POINT Z=80cm, R=75cm
FOR A SKEW DISTURBANCE
PLOT OF SKEW DISTURBANCE - REFLECTOR POINT

FIGURE 20



DEVIATION FROM N^3 SOLUTION AT REFLECTOR POINT Z=80cm, R=75cm
COMPARISON - SKEW DISTURBANCE - REFLECTOR POINT

FIGURE 21

APPENDIX C

COMPUTER PROGRAMMING CODES

APPENDIX C

COMPUTER PROGRAMMING CODES

I. DISCUSSION OF PROCEDURES

II. FED-2 LINEARIZED LISTING

III. FED-2 COMPACT Nxp LISTING

IV. DATA PROCESSOR LISTING

V. COMPACTING DEMONSTRATION LISTING WITH RESULTS

I. DISCUSSION

A. The FED-2 Programs

Both of the FED-2 programs consist of a small calling program which initiates the problem. The rest of the program is stored in the computer library. The advantages of a pre-compiled main program in time and convenience is clear, but the use of dummy dimension statements to pass data storage throughout the calling of the external subroutine DVOGER and its user-supplied subroutine appears as a trick not normally considered possible with the FORTRAN language.

When the original N^3 FED-2 program was written, every available device for reducing storage was attempted. Thus, where properties are normally summed over areas in the finite element method, the operations were carried out on the nodal points, since there are always fewer node points than elements. The new FED-2 programs continued these procedures not only to provide a valid comparison but also because this economizing technique is felt to be fundamentally sound even if unconventional. Employing this device, NFULEL is the number of node points associated with the core (fueled) region, whereas it would conventionally be the number of core (fueled) elements.

For these programs, three interest points IPT1, IPT2, and IPT3, may be selected for detailed investigation, since the purview of information produced by the printed output of these programs is quite large, making detailed study difficult. The time history of the neutron flux (PSI) at each of these interest points is recorded throughout the problem in an array BATCH. If it is desired to use this collection, it can be written onto a computer storage by the inclusion of additional JCL in the GO step. In this manner, data was saved for analysis by the data processing program used in this thesis to prepare the graphs of Figures 4 - 21.

An options chart and a schedule for FED-2 input data deck listing are included in this section in addition to a listing of the programs.

OPTION SELECTIONS FOR FED-2 PROGRAMS

NCOUNT

= 0 PREMULTIPLICATION BY $\psi_{t-\Delta t}$

= 1 PREMULTIPLICATION BY ψ TRIAL

MTH

= 1 DVOGER, Jacobian must be supplied

= 2 DVOGER, computes its own Jacobian

= 0 DVOGER, doesn't use Jacobian

= 5 CRANKO routine specified, DVOGER not activated

ERRVAL

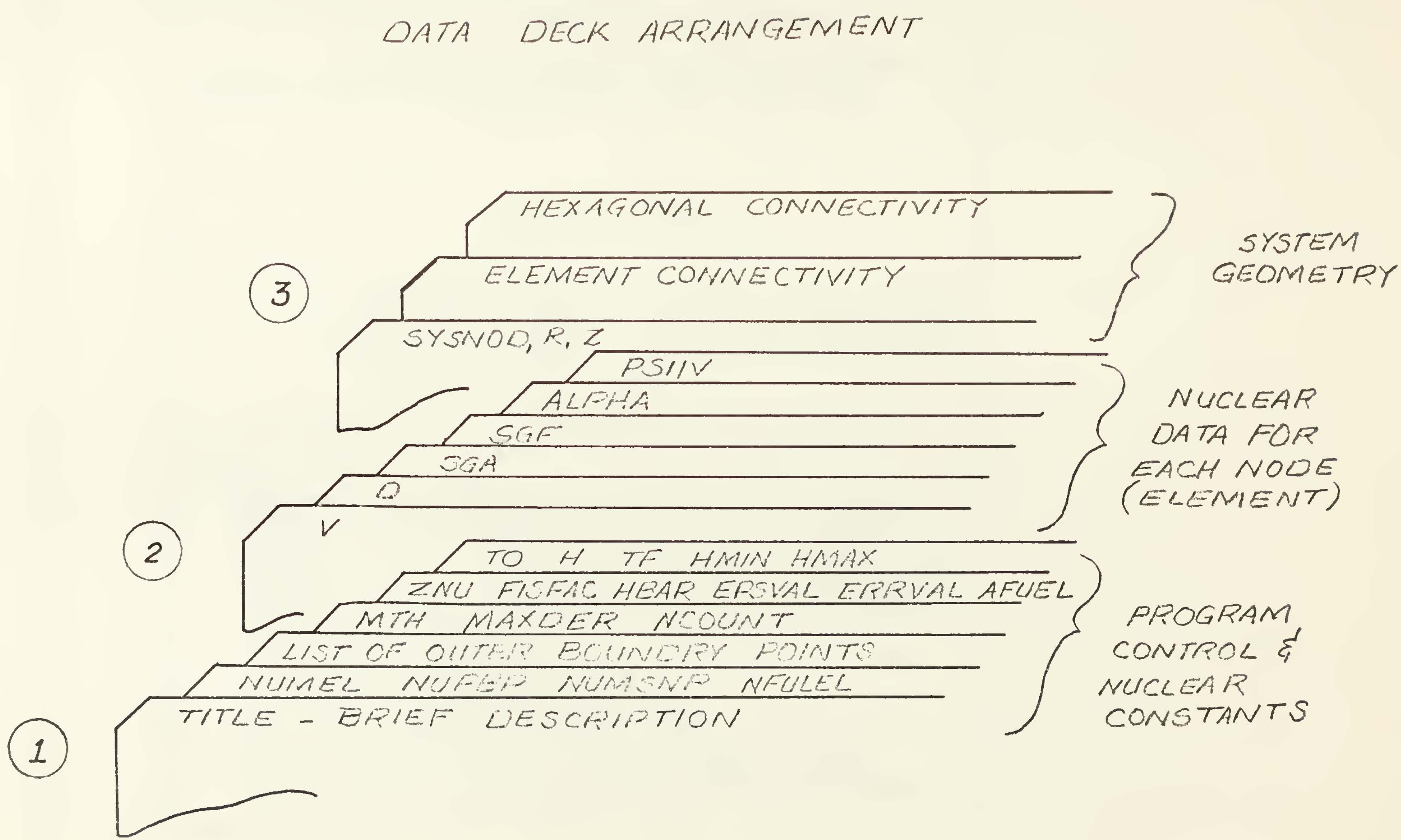
1- used internally by DVOGER, card input value has no effect

2- used by CRANKO, it is the convergence criterion for satisfactory ψ values

EPSVAL

1- convergence criterion for DVOGER

2- for CRANKO, sets the relative problem time difference required for a solution to be printed out


DATA DECK ARRANGEMENT
HEXAGONAL CONNECTIVITY
ELEMENT CONNECTIVITY
SYSNOD, R, Z
SYSTEM GEOMETRY
3
PSIIV
ALPHA
SGF
SGA
D
V
NUCLEAR DATA FOR EACH NODE (ELEMENT)
2
TO H TF HMIN HMAX
ZNU FISFAC HBAR ERSVAL ERRVAL AFUEL
MTH MAXDER NCOUNT
LIST OF OUTER BOUNDRY POINTS
NUMEL NUPBP NUMSNP NFULEL
TITLE - BRIEF DESCRIPTION
PROGRAM CONTROL & NUCLEAR CONSTANTS
1

```
C
C
C
C
C     ***********************************************************
C     *                                                         *
C     *                   PROGRAM FED-2-COMPACT                 *
C     *                                                         *
C     * FINITE ELEMENT SOLUTION OF NON-LINEAR REACTOR           *
C     * DYNAMICS IN TWO DIMENSIONAL SPACE WITH PRE-             *
C     * MULTIPLICATION TO REDUCE THE NON-LINEARITY.             *
C     *                                                         *
C     *                                                         *
C     *                                                         *
C     * SOLUTION METHODS                                        *
C     *                   1-DVOGER (IMSL)                       *
C     *                   2-CRANKO (INCLUDED)                   *
C     *                                                         *
C     * OPTIONS                                                 *
C     *       1- ALL OPTIONS AVAILABLE TO DVOGER ROUTINE        *
C     *          WITH SELECTION OF EITHER PREVIOUS TIME         *
C     *          OR CURRENT UPDATE                              *
C     *       2- CRANKO ROUTINE BASED ON CRANK-NICOLSON         *
C     *          FORMULATION AND SOLVED BY GAUSS-SEIDEL         *
C     *          ITERATION.                                     *
C     *                                                         *
C     *                                                         *
C     * INPUT PARAMETERS AND DIMENSIONING REQUIREMENTS.         *
C     *                                                         *
C     * SINGLE ITEMS                                            *
C     *    NUMSNP IS THE NUMBER OF SYSTEM NODAL POINTS.         *
C     *    NUMEL  IS THE NUMBER OF TRIANGULAR ELEMENTS.         *
C     *    NUPBP  IS THE NUMBER OF EXTERIOR BOUNDARY PTS.       *
C     *    NUFULEL IS THE NUMBER OF ELEMENTS IN THE CORE.       *
C     *    EPSVAL IS THE CONVERGENCE CRITERIA FOR DVOGER        *
C     *    ERRVAL IS THE CONVERGENCE CRITERIA FCR CRANKO        *
C     *    NCOUNT IS THE SELECTOR FOR UPDATE OR PREVIOUS        *
C     *       TIME  FOR DVOGER.                                 *
C     *    MTH IS THE OPTION SWITCH FOR CRANKO CR THE           *
C     *       SEVERAL DVOGER METHODS.                           *
C     *    TO IS THE PROBLEM START TIME                         *
C     *    TF IS THE PROBLEM FINISH TIME                        *
C     *    H  IS THE FIRST ATTEMPTED TIME INTERVAL              *
C     *    HMIN IS THE SMALLEST TIME INTERVAL                   *
C     *    HMAX IS THE LARGEST ALLOWED TIME INTERVAL            *
C     *    LCON IS THE MAX NUMBER OF NODE CONTRIBUTORS          *
C     *                                                         *
C     * THESE ARE THE INPUT REQUIREMENTS                        *
```

```
C        *    TITLE  NUMEL NUPBP NUMSNP NFUEL IUPBP MTH          *
C        *    MAXDR NCOUNT ZNU FISFAC HBAR EPSVAL ERRVAL         *
C        *    AFUEL TO H TF HMIN HMAX V D SGA SGF ALPHA          *
C        *    PSIIV SYSNOD R Z ELEMNT ELNOD ITYPE LCON MNOD      *
C        *                                                       *
C        * ITYPE=0  DENOTES CORE ELEMENTS                        *
C        * ITYPE=1  DENOTES BLANKET ELEMENTS                     *
C        *                                                       *
C        *  IPT1, IPT2, IPT3 ARE SELECTED INTEREST POINTS        *
C        *                                                       *
C        * ICON AND IPT1,IPT2,IPT3 ARE NOT ON DATA CARDS,        *
C          BUT ARE INSERTED DIRECTLY INTO THE CALLING PGM.    *
C        *                                                       *
C-----------------------------------------------------------------------
C
C
C
C
C         THESE ARE THE DIMENSIONING REQUIREMENTS.
C               PART(NUMSNP,LCON),BIGA(NUMSNP,LCON),
C         BIGAB(NUMSNP,LCON),BIGC(NUMSNP,LCON),BIGCC(NUMSNP,LCON),
C         MNOD(NUMSNP,LCON),V(NUMSNP),D(NUMSNP),SGA(NUMSNP),
C         SGF(NUMSNP),ALPHA(NUMSNP),PSIIV(NUMSNP),PDPSI(NUMSNP)
C                ZLMBDA(NUMSNP),  VLMBDA(NUMSNP),ZOMEGA(NUMSNP),
C         RHO(NUMSNP),SYSNOD(NUMSNP),R(NUMSNP),Z(NUMSNP),
C         ELEMNT(NUMEL),ELNOD(NUMEL,3),ITYPE(NUMEL),R1(NUMEL),
C         Z1(NUMEL),R2(NUMEL),Z2(NUMEL),R3(NUMEL),Z3(NUMEL)
C               A(NUMEL,3),B(NUMEL,3),PW(NWK),BATCH(400,4),
C         ERROR(NUMSNP),CM(NUMEL,3,3,3),ALFA(3,3),PSTEP(NUMSNP),
C        BIGD(NUMSNP,LCON),BIGE(NUMSNP,LCON),ESTR(NUMSNP),VD(NUMSNP),
C        YMAX(NUMSNP),IUPBP(NUPBP),PSI(8,NUMSNP),DPSI(NUMSNP),
C        AREA(NUMEL),PV(NUMSNP),HOLD(NUMSNP)
C-----------------------------------------------------------------------
C
C
C
C
C--------- DECLARATION STATEMENTS --------------------------------------
C
C     NONLINEAR SUPERCRITICAL REACTOR
C     PROMPT FEEDBACK
C     FULLY REFLECTED SYSTEM
C     SPACE-DEPENDENT NEUTRONIC PROPERTIES
C
C
C*********************************************************************@
C
      INTEGER*4 SYSNOD,ELEMNT,ELNOD
```

```
C
      DIMENSICN TITLE(20)
C
C
      DIMENSION PART(132,7),BIGA(132,7),BIGAB(132,7),BIGC(132,7),
     1   BIGCC(132,7),MNOD(132,7),V(132),D(132),SGA(132),SGF(132),
     2   ALPHA(132),PSIIV(132),PDPSI(132),ZLMBDA(132),VLMBDA(132),
     3   ZOMEGA(132),RHO(132),SYSNOD(132),R(132),Z(132),ELEMNT(220),
     4   ELNOD(220,3),ITYPE(220),R1(220),Z1(220),R2(220),Z2(220),
     5   R3(220),Z3(220),A(220,3),PW(2244),BATCH(400,4),ERROR(132),
     6   CM(220,3,3,3),PSTEP(132),BIGD(132,7),BIGE(132,7),ESTR(132),
     7   VD(132),YMAX(132),IUPBP(22),PSI(8,132),AREA(220),PV(132),
     8   B(220,3),ASTR(132,7),HOLD(132)
C
C
      READ(5,998) TITLE
      READ(5,10) NUMEL,NUPBP,NUMSNP,NFULEL
  998 FCRMAT(20A4)
   10 FORMAT(8I10)
C
C        FOR MTH=1  NWK MUST BE DIMENSIONED AS NUMSNP*(NUMSNP+17)
      NWK=NUMSNP*17
      LCCN=7
C
      IPT1=1
      IPT2=41
      IFT3=91
C
C
C     *                                                              *
           CALL FEDMAN (NUMEL,NUPBP,NUMSNP,NFULEL,LCON,NWK,TITLE,
     1   IPT1,IPT2,IPT3,PART,BIGA,BIGAB,BIGC,BIGCC,MNOD,V,D,SGA,SGF,
     2   ALPHA,PSIIV,PDPSI,ZLMBDA,VLMBDA,ZOMEGA,RHO,SYSNOD,R,Z,ELEMNT,
     3   ELNOD,ITYPE,R1,Z1,R2,Z2,R3,Z3,A,B,PW,BATCH,ERROR,CM,ALFA,
     4   PSTEP,BIGD,BIGE,ASTR,ESTR,VD,YMAX,DPSI,  IUPBP,PSI,AREA,PV,
     9   HOLD)
C     *                                                              *
      STOP
      END
      SUBROUTINE FEDMAN (NUMEL,NUPBP,NUMSNP,NFULEL,LCON,NWK,TITLE,
     1   IPT1,IPT2,IPT3,PART,BIGA,BIGAB,BIGC,BIGCC,MNOD,V,D,SGA,SGF,
     2   ALPHA,PSIIV,PDPSI,ZLMBDA,VLMBDA,ZOMEGA,RHO,SYSNOD,R,Z,ELEMNT,
     3   ELNOD,ITYPE,R1,Z1,R2,Z2,R3,Z3,A,B,PW,BATCH,ERROR,CM,ALFA,
     4   PSTEP,BIGD,BIGE,ASTR,ESTR,VD,YMAX,DPSI,  IUPBP,PSI,AREA,PV,
     9   HOLD)
C     *                                                              *
C     *                                                              *
C     *          FINITE ELEMENT SOLUTION OF NONLINEAR                *
```

```
C      *          REACTOR DYNAMICS IN TWO-DIMENTIONAL SPACE          *
C      *                                                             *
C      *                                                             *
       INTEGER*4 SYSNOD,ELEMNT,ELNOD
C
       DIMENSION TITLE(20)
C
       DIMENSION PART(NUMSNP,LCON),BIGA(NUMSNP,LCON),
      1    BIGAB(NUMSNP,LCON),BIGC(NUMSNP,LCON),BIGCC(NUMSNP,LCON),
      2    MNOD(NUMSNP,LCON),V(NUMSNP),D(NUMSNP),SGA(NUMSNP),
      3    SGF(NUMSNP),ALPHA(NUMSNP),PSIIV(NUMSNP),PDPSI(NUMSNP)
       DIMENSION  ZLMBDA(NUMSNP), VLMBDA(NUMSNP),ZOMEGA(NUMSNP),
      1    RHO(NUMSNP),SYSNOD(NUMSNP),R(NUMSNP),Z(NUMSNP),
      2    ELEMNT(NUMEL),ELNOD(NUMEL,3),ITYPE(NUMEL),R1(NUMEL),
      3    Z1(NUMEL),R2(NUMEL),Z2(NUMEL),R3(NUMEL),Z3(NUMEL)
       DIMENSION A(NUMEL,3),B(NUMEL,3),PW(NWK),BATCH(400,4),
      1    ERROR(NUMSNP),CM(NUMEL,3,3,3),ALFA(3,3),PSTEP(NUMSNP),
      2   BIGD(NUMSNP,LCON),BIGE(NUMSNP,LCON),ESTR(NUMSNP),VD(NUMSNP),
      3   YMAX(NUMSNP),IUPBP(NUPBP),PSI(8,NUMSNP),DPSI(NUMSNP),
      4   AREA(NUMEL),PV(NUMSNP),HOLD(NUMSNP)
C
       DO 05 I=1,NUMSNP
       ERROR(I)=0.
        PDPSI(I)=0.
       YMAX(I)=1.
    05 CONTINUE
C
C
       NUF=NUMSNP/5
       XNP=NUMSNP
       XNUF=XNP/5.
       IF ((XNUF-FLOAT(NUF)) .LT. 1.)  NUF=NUF+1
C
C
C   GENERAL PROBLEM DATA
C
       READ(5,12) (IUPBP(I),I=1,NUPBP)
       READ(5,10) MTH,MAXDER,NCOUNT
    10 FORMAT(8I10)
    12 FORMAT(16I5)
C
       WRITE(6,11) NUMEL,NUPBP,NUMSNP,NFULEL
    11 FORMAT(//2X,'# OF ELEMENTS=',I5//2X,'# OF UPPER BDRY NODAL POINTS=
      1',I5//2X,'# OF SYSTEM NODAL POINTS=',I5//2X,'# OF FUEL ELEMENTS=',
      2I5)
C
       READ(5,15) ZNU,FISFAC,HBAR,EPSVAL,ERRVAL,AFUEL
       READ(5,19)TO,H,TF,HMIN,HMAX
```

```
      READ(5,15) (V(I),I=1,NUMSNP)
      READ(5,15) (D(I),I=1,NUMSNP)
      READ(5,15) (SGA(I),I=1,NUMSNP)
      READ(5,15) (SGF(I),I=1,NUMSNP)
      READ(5,15) (ALPHA(I),I=1,NUMSNP)
      READ(5,18) (PSIIV(I),I=1,NUMSNP)
   15 FORMAT(8G10.5)
C
C
C     CALCULATE PHYSICAL CONSTANTS FROM NUCLEAR DATA
C
      PI= 3.1415927
C
      DO 25 I=1,NUMSNP
      VD(I)=V(I)*D(I)
      ZLMBDA(I)=ZNU*SGF(I)/SGA(I)-1.
      VLMBDA(I)=V(I)*ZLMBDA(I)*SGA(I)
      ZOMEGA(I)=V(I)*SGA(I)*ALPHA(I)*FISFAC*SGF(I)/HBAR
      RHO(I)=ZNU*(SGF(I)-0.00621835)/(SGA(I)*0.003297)
      IF (I .GT. NFULEL) RHO(I)=.0
   25 CONTINUE
C
C
   18 FORMAT(5E15.4)
   19 FORMAT(2E20.4,4G10.5)
  998 FORMAT(20A4)
C
C
C
C
C
C--------- GEOMETRY OF SYSTEM NODAL POINTS -------------------------------------
C
      WRITE(6,40)
   40 FORMAT    (///1X,'GEOMETRY OF SYSTEM NODAL POINTS'///)
C
      READ(5,41) (SYSNOD(I),R(I),Z(I),I=1,NUMSNP)
   41 FORMAT(I10,2G15.8)
C
C
      WRITE      (6,60) (SYSNOD(I),R(I),Z(I),I=1,NUMSNP)
   60 FORMAT (1X,'SYSTEM NODAL POINT NO. ',I3,5X,'R COORD.= ',F10.6,5X,'
     1Z COORD.= ',F10.6)
C
C
C
C*****************************************************************************
C CORRESPONDENCE TABLE BETWEEN SYSTEM AND ELEMENT NODAL POINTS
```

```
C*********************************************************************@
C
C
      WRITE(6,73)
   73 FORMAT(//2X,'CONNECTIVITY MATRIX',//,' EL #',33X,'TYPE'//)
      DO 75 I=1,NUMEL
      READ(5,76) ELEMNT(I),ELNOD(I,1),ELNOD(I,2),ELNOD(I,3),ITYPE(I)
      WRITE(6,77) ELEMNT(I),ELNOD(I,1),ELNOD(I,2),ELNOD(I,3),ITYPE(I)
   75 CONTINUE
   76 FORMAT (5I10)
   77 FORMAT (2X,I3,4I10)
C
C
C     READ AND WRITE THE HEXAGONAL CONNECTIVITY MATRIX.
C     LCON IS THE MAX NR OF NODAL POINT CONTRIBUTORS.
      DO 776 J=1,NUMSNP
      READ (5,777) (MNOD(J,K),K=1,LCON)
  776 CONTINUE
  777 FORMAT  (9(2X,I3))
      DO 775 J=1,NUMSNP
      WRITE (6,777) (MNOD(J,K),K=1,LCON)
  775 CONTINUE
C
C
C
C     WHEN REQUIRED THE INITIAL DERIVATIVES ARE READ IN HERE,
C     INSERTED AT THE END OF THE DATA DECK IN FORMAT 771
C
C     READ AND WRITE THE INITIAL DIRRIVATIVES OF THE NODE POINTS.
C     READ (5,771) (PDPSI(J),J=1,NUMSNP)
  771 FORMAT (4E15.5)
      WRITE (6,771) (PDPSI(J),J=1,NUMSNP)
C
C
C
C*********************************************************************@
C CALCULATE GEOMETRY FOR EACH ELEMENT
C*********************************************************************@
C
      WRITE(6,110)
  110 FORMAT (1X///1X,'GEOMETRY CALCULATIONS FOR EACH ELEMENT')
C
C---------------------------------------------------------------------
C LOAD SYSTEM NODE COORDINATES (R,Z) INTO
C ELEMENT NODE COORDINATES (R1,R2,R3,Z1,Z2,Z3)
C
      WRITE(6,95)
   95 FORMAT (1X///1X,'ELEMENT',5X,'R1',9X,'Z1',9X,'R2',9X,'Z2',9X,'R3',
```

```
     19X,'Z3'/)
C
      DO 100 I=1,NUMEL
C
      J=ELNOD(I,1)
      R1(I)=R(J)
      Z1(I)=Z(J)
C
      K=ELNOD(I,2)
      R2(I)=R(K)
      Z2(I)=Z(K)
C
      L=ELNOD(I,3)
      R3(I)=R(L)
      Z3(I)=Z(L)
C
      WRITE      (6,105) I,R1(I),Z1(I),R2(I),Z2(I),R3(I),Z3(I)
C
  100 CONTINUE
  105 FORMAT (3X,I3,3X,   7(F10.6,1X))
C
C COMPUTE A1,A2,A3,B1,B2,B3,AREA FOR EACH ELEMENT
C---------------------------------------------------------------------------
C
      WRITE      (6,120)
  120 FORMAT (1X///1X,'ELEMENT',5X,'A1',9X,'A2',9X,'A3',9X,'B1',9X,'B2',
     19X,'B3',8X,'AREA'/)
C
      DO 140 I=1,NUMEL
      A(I,1)=R3(I)-R2(I)
      A(I,2)=R1(I)-R3(I)
      A(I,3)=R2(I)-R1(I)
      B(I,1)=Z2(I)-Z3(I)
      B(I,2)=Z3(I)-Z1(I)
      B(I,3)=Z1(I)-Z2(I)
      AREA(I)=0.5*(A(I,2)*B(I,1)-A(I,1)*B(I,2))
C
      WRITE(6,130) I,A(I,1),A(I,2),A(I,3),B(I,1),B(I,2),B(I,3),
     1AREA(I)
  140 CONTINUE
C
  130 FORMAT (3X,I3,3X,   7(F12.7,1X))
C
C
C
C--------- ZERO THE BIGA,     BIGC AND BIGAB MATRICES-----------------------
C
C
```

```
C
      DO 194  KK=1,NUMSNP
      DO 193 II=1,LCON
      BIGA(KK,II)=0.
      BIGAB(KK,II)=0.0
      BIGC(KK,II)=.0
      PART(KK,II)=.0
  193 CONTINUE
  194 CONTINUE
C
C
C******************************************************* ****************
C
C
C-----------------------------------------------------------------------
C     PEPSI CALCULATES THE BIGA MATRIX AND PART OF THE BIGAB
C-----------------------------------------------------------------------
C
      CALL PEPSI (NUMEL,NUMSNP,R1,R2,R3,AREA,ELNOD,LCON,MNOD,
     1     BIGA,BIGAB,VLMBDA)
C
  195 FORMAT(//5X,'BIGA MATRIX',//)
      WRITE(6,195)
      WRITE(6,778)((BIGA(I,J),J=1,LCON) ,I=1,NUMSNP)
  778 FORMAT (2X,7(E14.5))
  196 FORMAT(//5X,'BIGAB MATRIX FROM PEPSI, NO DEL**2 CONTRIBUTION',//)
      WRITE(6,778) ((BIGAB(I,J),J=1,LCON),I=1,NUMSNP)
C
C
C-----------------------------------------------------------------------
C     CRUISE CALCULATES THE BIGAB MATRIX
C-----------------------------------------------------------------------
C
      CALL CRUISE (NUMEL,NUMSNP,LCON,R1,R2,R3,AREA,ELNOD,
     1     BIGAB,MNOD,A,B,VD)
C
      WRITE(6,197)
  197 FORMAT(//5X,'BIGAB MATRIX FROM CRUISE, WITH DEL**2 TERM',//)
      WRITE(6,778) ((BIGAB(I,J),J=1,LCON),I=1,NUMSNP)
C
C-----------------------------------------------------------------------
C
C
C
      DO 255 I=1,NUPBP
      II=IUPBP(I)
      DO 251 J=1,NUMSNP
      BIGAB(II,1)=.0
```

```
      BIGA(II,1)=1.
      DC 241  MX=2,LCON
      BIGA(II,MX)=.0
      BIGAB(II,MX)=.0
  241 CONTINUE
  251 CONTINUE
  255 CONTINUE
  753 FORMAT (///10X,'AB AND A MATRICIES AFTER BOUNDRY POINTS'/)
C
C
      WRITE (6,753)
C
      WRITE(6,778) ((BIGAB(I,J),J=1,LCON),I=1,NUMSNP)
C
      WRITE(6,778)((BIGA(I,J),J=1,LCON) ,I=1,NUMSNP)
C
C
C
C
C     WRITE OUT NUCLEAR DATA
C
      WRITE(6,3105) ZNU,FISFAC,HBAR,EPSVAL,ERRVAL,AFUEL
 3105 FORMAT(//5X,'NUCLEAR DATA'/2X,'ZNU=',G12.6/2X,'FISFAC=',G12.6/2X,
     1 'HBAR=',G12.6/2X,'EPSVAL=',G12.6/2X,'ERRVAL=',G12.6/2X,'AFUEL=',
     2 G12.6)
      WRITE(6,3110)
 3110 FORMAT(1H0,2X,'NODE',6X,'D',8X,'SGA',9X,'SGF',8X,'ALPHA',9X,'VD',
     1 7X,'ZLMBDA',7X,'VLMBDA',7X,'ZOMEGA',8X,'V',9X,'RHO($)')
      DO 3115 I=1,NUMSNP
      WRITE(6,3120) (I,D(I),SGA(I),SGF(I),ALPHA(I),VD(I),ZLMBDA(I),
     1 VLMBDA(I),ZOMEGA(I),V(I),RHO(I))
 3115 CONTINUE
 3120 FORMAT(2X,I4,10(1PE12.4))
C
C
C     WRITE THE INITIAL VALUES OF THE FLUX
      WRITE (6,771) (PSIIV(J),J=1,NUMSNP)
C
C
C     ************************************************************
C
C--------- INITIALIZE ARGUMENTS OF DVOGER -------------------------------------
C
      NIT=0
      T=TO
      JSTART=0
      EPS= EPSVAL
      STAR=EPSVAL
      IER=0
```

```
      PTIME= H
C
      WRITE(6,999) TITLE
  999 FORMAT(1H1,20A4)
      IF (MTH .EQ. 0)  WRITE (6,33)
      IF (MTH .EQ. 1)  WRITE (6,31)
      IF (MTH .EQ. 5)  WRITE (6,35)
   31 FORMAT (10X,'THIS PGM UTILIZES GEERS METHOD IN DOVGER')
   33 FORMAT (10X,'THIS PGM UTILIZES ADAMS PERDICTOR-CORRECTOR ',
     1   'METHOD IN DVOGER')
   35 FORMAT (10X,'THIS PGM UTILIZES CRANK-NICOLSCN METHOD TO SET',
     1   'UP A SET OF SIMULTANEOUS LINEAR EQNS WHICH ARE THEN'
     2   ,/10X,'SOLVED BY SUCCESSIVE ITERATION WITH IMPROVEMENT.')
C
      WRITE(6,317)
  317 FCRMAT  (//10X,'INITIAL ARGUMENTS',//)
      IF (MTH .EQ. 5) GO TO 319
      WRITE(6,318) T,MTH,MAXDER,JSTART,H,HMIN,HMAX,EPS,NCOUNT
  318 FORMAT(2X,'T=',G10.4,2X,'MTH=',I4,2X,'MAXDER=',I4,2X,'JSTART=',I4,
     12X,'H=',G10.4,2X,'HMIN=',G10.4,2X,'HMAX=',G10.4,2X,'EPS=',G10.4,2X
     2,'NCOUNT=',I2,/)
  319 CCNTINUE
C
C
      DO 320 I=1,NUMSNP
      ERROR(I)= EKRVAL
      YMAX(I)=1.
      PSI(1,I)=PSIIV(I)
  320 CONTINUE
C
      DO 321 I=1,NUPBP
      J=IUPBP(I)
      PSI(1,J)=0.0
  321 CONTINUE
C
      NUMEQ=NUMSNP-NUPBP
C
C
C
C     CONSTRUCT THE BIGC MATRIX ON THE ELEMENT LEVEL
C-----------------------------------------------------------------------
C
      DO 200 L=1,NUMEL
C
      DO 583 I=1,3
      DO 582 J=1,3
      DO 581 K=1,3
      CM(L,I,J,K) =0.
```

```
  581 CONTINUE
  582 CONTINUE
  583 CONTINUE
      LL=ITYPE(L)
      IF (LL .NE. 0) GO TO 200
C
      CC=PI*AREA(L)/180.
C
      CM(L,3,3,3)=CC*(6.0*R1(L)+6.0*R2(L)+24.*R3(L))
      CM(L,3,3,2)=CC*(2.0*R1(L)+4.0*R2(L)+6.0*R3(L))
      CM(L,3,3,1)=CC*(4.0*R1(L)+2.0*R2(L)+6.0*R3(L))
      CM(L,3,2,3)=CC*(2.0*R1(L)+4.0*R2(L)+6.0*R3(L))
      CM(L,3,2,2)=CC*(2.0*R1(L)+6.0*R2(L)+4.0*R3(L))
      CM(L,3,2,1)=CC*(2.0*R1(L)+2.0*R2(L)+2.0*R3(L))
      CM(L,3,1,3)=CC*(4.0*R1(L)+2.0*R2(L)+6.0*R3(L))
      CM(L,3,1,2)=CC*(2.0*R1(L)+2.0*R2(L)+2.0*R3(L))
      CM(L,3,1,1)=CC*(6.0*R1(L)+2.0*R2(L)+4.0*R3(L))
      CM(L,2,3,3)=CC*(2.0*R1(L)+4.0*R2(L)+6.0*R3(L))
      CM(L,2,3,2)=CC*(2.0*R1(L)+6.0*R2(L)+4.0*R3(L))
      CM(L,2,3,1)=CC*(2.0*R1(L)+2.0*R2(L)+2.0*R3(L))
      CM(L,2,2,3)=CC*(2.0*R1(L)+6.0*R2(L)+4.0*R3(L))
      CM(L,2,2,2)=CC*(6.0*R1(L)+24.*R2(L)+6.0*R3(L))
      CM(L,2,2,1)=CC*(4.0*R1(L)+6.0*R2(L)+2.0*R3(L))
      CM(L,2,1,3)=CC*(2.0*R1(L)+2.0*R2(L)+2.0*R3(L))
      CM(L,2,1,2)=CC*(4.0*R1(L)+6.0*R2(L)+2.0*R3(L))
      CM(L,2,1,1)=CC*(6.0*R1(L)+4.0*R2(L)+2.0*R3(L))
      CM(L,1,3,3)=CC*(4.0*R1(L)+2.0*R2(L)+6.0*R3(L))
      CM(L,1,3,2)=CC*(2.0*R1(L)+2.0*R2(L)+2.0*R3(L))
      CM(L,1,3,1)=CC*(6.0*R1(L)+2.0*R2(L)+4.0*R3(L))
      CM(L,1,2,3)=CC*(2.0*R1(L)+2.0*R2(L)+2.0*R3(L))
      CM(L,1,2,2)=CC*(4.0*R1(L)+6.0*R2(L)+2.0*R3(L))
      CM(L,1,2,1)=CC*(6.0*R1(L)+4.0*R2(L)+2.0*R3(L))
      CM(L,1,1,3)=CC*(6.0*R1(L)+2.0*R2(L)+4.0*R3(L))
      CM(L,1,1,2)=CC*(6.0*R1(L)+4.0*R2(L)+2.0*R3(L))
      CM(L,1,1,1)=CC*(24.*R1(L)+6.0*R2(L)+6.0*R3(L))
C
  200 CONTINUE
C
  151 CONTINUE
C
C
      IF (MTH .EQ. 5) CALL CRANKO (NUMEL,NUMSNP,NUPBP,LCON,H,TO,
     1    ERRVAL,EPSVAL,IUPBP,PSIIV,ITYPE,ELNOD,CM,MNOD,ZOMEGA,
     2 BIGAB,BIGA,IPT1,IPT2,IPT3,PV,PSTEP,BIGD,BIGE,ASTR,ESTR,
     3   BIGC,BIGCC,HOLD,TF)
C
C
      EXTERNAL YVETTE
```

```
C
      CALL YVET (NUMEL,NUMEQ,NUMSNP,LCON,IUPBP,PSIIV,NUPBP,BIGC,
     1    ITYPE,ELNOD,CM,BIGA,BIGAB,MNOD,BIGCC,PART,NWK,PSI,PW,DPSI,
     2   PDPSI,HOLD,DPT,PV,NCOUNT,ZOMEGA)
C
  351 CONTINUE
C
      IF (NCOUNT .EQ. 1) GO TO 312
C
C*********************************************************************************
C          FIXES THE NEW BIGCC BY PREMULTIPLYING BY PSI(T-1)
C*********************************************************************************
C
      DO 210 L=1,NUMEL
      LL=ITYPE(L)
      IF(LL .NE. 0) GO TO 210
      N1=ELNOD(L,1)
      N2=ELNOD(L,2)
      N3=ELNOD(L,3)
      DO 88 J=1,3
      DO 90 N=1,3
      ALFA(N,J)=PSIIV(N1)*CM(L,N,J,1)+PSIIV(N2)*CM(L,N,J,2)
     1   +PSIIV(N3)*CM(L,N,J,3)
   90 CONTINUE
   88 CONTINUE
      DO 150 K=1,3
      KK=ELNOD(L,K)
      DO 180 I=1,3
      II=ELNOD(L,I)
      DO 91 MX=1,LCON
      NOW =MX
      MM     =MNOD(KK,MX)
      IF (MM      .EQ. II) GO TO 92
   91 CONTINUE
   92 CONTINUE
      BIGC(KK,NOW)=BIGC(KK,NOW)+ALFA(K,I)*ZOMEGA(KK)
  180 CONTINUE
  150 CONTINUE
  210 CONTINUE
C     WRITE(6,755)
  755 FORMAT(///10X,' BIGC MATRIX BEFORE BOUNDRY POINTS'/)
C     WRITE(6,778)((BIGC(I,J),J=1,LCON) ,I=1,NUMSNP)
C
C          INSERTION OF BOUNDARY POINTS
      DO 285 I=1,NUPBP
      II=IUPBP(I)
      DO 291 MX=1,LCON
      BIGC(II,MX)=.0
```

```
  291 CONTINUE
  285 CCNTINUE
C     WRITE(6,757)
  757 FORMAT(///10X,'BIGC MATRIX WITH BOUNDARY POINTS'/)
C     WRITE(6,778) ((BIGC(I,J),J=1,7),I=1,NUMSNP)
C
C
      DO 310  I=1,NUMSNP
      DO 310  J=1,LCON
      BIGCC(I,J)=BIGAB(I,J)-BIGC(I,J)
      PART(I,J)=BIGAB(I,J)-2.*BIGC(I,J)
  310 CONTINUE
C     WRITE(6,759)
  759 FORMAT (///10X,'BIGCC MATRIX'/)
C     WRITE(6,778) ((BIGCC(I,J),J=1,LCON),I=1,NUMSNP)
C
C
C
  312 CONTINUE
C
C
C          *************************************************************
C          * SOLVE NON-LINEAR FLUX EQUATIONS WITH SUBROUTINE DVOGER *
C          *************************************************************
C
  350 CALL DVOGER(YVETTE,PSI,T,NUMSNP,MTH,MAXDER,JSTART,H,HMIN,HMAX,EPS,
     1YMAX,ERROR,PW,IER)
C
C
      IF(IER.EQ.0) GO TO 3540
      JSTART=-1
      H=HMIN*.1
      HMIN=H*.1
      GO TO 350
C
 3540 CCNTINUE
      JSTART=1
      IF((T-PTIME)/PTIME .LT. STAR)  GO TO 362
      PTIME= T
      WRITE(6,3510)T,H,JSTART,IER
 3510 FORMAT(//2X,'T=',G12.6,2X,'H=',G12.6,2X,'JSTART=',I4,2X,'IER=',I4)
      WRITE (6,354)
  354 FORMAT(/(5(4X,'NODE',7X,'PSI',4X)))
C
C
      NIT= 1 + NIT
      IF (NIT .GT. 400)  GO TO 377
      BATCH (NIT,1)=T
```

```
      BATCH(NIT,2)=PV(IPT1)
      BATCH(NIT,3)=PV(IPT2)
      BATCH(NIT,4)=PV(IPT3)
  377 CONTINUE
C
      DO 355 I=1,NUF
      I1=I+NUF
      I2=I+2*NUF
      I3=I+3*NUF
      I4=I+4*NUF
      WRITE(6,356) (I,PSI(1,I),I1,PSI(1,I1),I2,PSI(1,I2),I3,PSI(1,I3),I4
     1,PSI(1,I4))
  355 CONTINUE
  356 FORMAT(5(4X,I3,3X,1PE12.4))
C
      DO 360 I=1,NUMSNP
        R(I)=PSI(2,I)/H
  360 CONTINUE
C
      WRITE(6,357)
  357 FORMAT(/(5(4X,'NODE',7X,'DPSI',3X)))
      DO 359 I=1,NUF
      I1=I+NUF
      I2=I+2*NUF
      I3=I+3*NUF
      I4=I+4*NUF
      WRITE(6,356) (I,R(I),I1,R(I1),I2,R(I2),I3,R(I3),I4,R(I4))
  359 CONTINUE
  362 CONTINUE
C
      DO 27 MN=1,NUMSNP
      PSIIV(MN)=PSI(1,MN)
      DO 28 I=1,LCON
      BIGC(MN,I)=.0
   28 CONTINUE
   27 CONTINUE
C
      IF (T .LT. TF) GO TO 351
C
C
  399 FORMAT (10X,I4)
      WRITE (6,399) NIT
      WRITE (3) BATCH
      WRITE (4) BATCH
      STOP
      END
      SUBROUTINE PEPSI (NUMEL,NUMSNP,R1,R2,R3,AREA,ELNOD,LCON,MNOD,
     1   BIGA,BIGAB,VLMBDA)
```

```
C
C*******************************************************************************
C
C     PEPSI CALCULATES THE BIGA MATRIX AND PART OF THE BIGAB
C*******************************************************************************
C
      INTEGER*4 ELNOD
C
      DIMENSION  R1(NUMEL),R2(NUMEL),R3(NUMEL),BIGA(NUMSNP,LCON),
     1    BIGAB(NUMSNP,LCON),MNOD(NUMSNP,LCON),AMATRX(3,3),
     2     VLMBDA(NUMSNP),AREA(NUMEL),ELNOD(NUMEL,3)
C
      PI=3.1415927
C
C CALCULATE THE 3X3 D(I,J) MATRIX FOR ELEMENT L
C
C
      DO 200 L=1,NUMEL
      COEFFA= (PI/30.0) * AREA(L)
C
      AMATRX(1,1)=COEFFA   *(6.0*R1(L)+2.0*R2(L)+2.0*R3(L))
      AMATRX(1,2)=COEFFA   *(2.0*R1(L)+2.0*R2(L)+R3(L))
      AMATRX(1,3)=COEFFA   *(2.0*R1(L)+R2(L)+2.0*R3(L))
      AMATRX(2,1)=AMATRX(1,2)
      AMATRX(2,2)=COEFFA   *(2.0*R1(L)+6.0*R2(L)+2.0*R3(L))
      AMATRX(2,3)=COEFFA   *(R1(L)+2.0*R2(L)+2.0*R3(L))
      AMATRX(3,1)=AMATRX(1,3)
      AMATRX(3,2)=AMATRX(2,3)
      AMATRX(3,3)=COEFFA   *(2.0*R1(L)+2.0*R2(L)+6.0*R3(L))
C
C-------------------------------------------------------------------------------
C STORE ELEMENT MATRIX, AMATRX, INTO SYSTEM MATRIX, BIGA
C-------------------------------------------------------------------------------
C
      DO 20 K=1,3
      KK=ELNOD(L,K)
      DO 10 I=1,3
      II=ELNOD(L,I)
      DO 91 MX=1,LCON
      NOW =MX
      MM    =MNOD(KK,MX)
      IF (MM     .EQ. II) GO TO 92
   91 CONTINUE
   92 CONTINUE
      BIGA(KK,NOW)=BIGA(KK,NOW)+AMATRX(K,I)
      BIGAB(KK,NOW)=BIGAB(KK,NOW)+VLMBDA(KK)*AMATRX(K,I)
   10 CONTINUE
   20 CONTINUE
```

```
C
  200 CONTINUE
C
      RETURN
      END
      SUBROUTINE CRUISE (NUMEL,NUMSNP,LCON,R1,R2,R3,AREA,ELNOD,BIGAB,
     1    MNOD,A,B,VD)
C     *                                                        *
C
C     CRUISE CALCULATES THE BIGAB MATRIX
C-----------------------------------------------------------------------
C
      INTEGER*4 ELNOD
C
      DIMENSION R1(NUMEL),R2(NUMEL),R3(NUMEL),AREA(NUMEL),
     1    MNOD(NUMSNP,LCON),BIGAB(NUMSNP,LCON),BMATRX(3,3),
     2   ELNOD(NUMEL,3),A(NUMEL,3),B(NUMEL,3),VD(NUMSNP)
C
      PI=3.1415927
C
C
C CALCULATE THE 3X3 B(I,J) MATRIX FOR ELEMENT L
C-----------------------------------------------------------------------
C
      DC 200 L=1,NUMEL
C
      COEFFB= PI / (6.0 * AREA(L) )
C
      BMATRX(1,1)=COEFFB   *(-R1(L)*(2.0*B(L,1)**2+2.0*A(L,1)**2)-R2(L)*
     1(B(L,1)*B(L,2)+B(L,1)**2+A(L,1)*A(L,2)+A(L,1)**2)-R3(L)*(B(L,1)*
     2B(L,3)+B(L,1)**2+A(L,1)*A(L,3)+A(L,1)**2)+2.0*AREA(L)*B(L,1))
C
      BMATRX(1,2)=COEFFB   *(-R1(L)*(2.0*B(L,1)*B(L,2)+2.0*A(L,1)*
     1A(L,2))-R2(L)*(B(L,2)**2+B(L,1)*B(L,2)+A(L,2)**2+A(L,1)*A(L,2))-
     2R3(L)*(B(L,2)*B(L,3)+B(L,1)*B(L,2)+A(L,2)*A(L,3)+A(L,1)*A(L,2))+
     32.0*AREA(L)*B(L,2))
C
      BMATRX(1,3)=COEFFB   *(-R1(L)*(2.0*B(L,1)*B(L,3)+2.0*A(L,1)*
     1A(L,3))-R2(L)*(B(L,2)*B(L,3)+B(L,1)*B(L,3)+A(L,2)*A(L,3)+A(L,1)*
     2A(L,3))-R3(L)*(B(L,3)**2+B(L,1)*B(L,3)+A(L,3)**2+A(L,1)*A(L,3))+
     32.0*AREA(L)*B(L,3))
C
      BMATRX(2,1)=BMATRX(1,2)
C
      BMATRX(2,2)=COEFFB   *(-R1(L)*(B(L,2)**2+B(L,1)*B(L,2)+A(L,2)**2+
     1A(L,1)*A(L,2))-R2(L)*(2.0*B(L,2)**2+2.0*A(L,2)**2)-R3(L)*(B(L,3)*
     2B(L,2)+B(L,2)**2+A(L,3)*A(L,2)+A(L,2)**2)+2.0*AREA(L)*B(L,2))
C
```

```
      BMATRX(2,3)=COEFFB    *(-R1(L)*(B(L,2)*B(L,3)+B(L,1)*B(L,3)+A(L,2)*
     1A(L,3)+A(L,1)*A(L,3))-R2(L)*(2.0*B(L,2)*B(L,3)+2.0*A(L,2)*A(L,3))-
     2R3(L)*(B(L,3)**2+B(L,2)*B(L,3)+A(L,3)**2+A(L,2)*A(L,3))+
     32.0*AREA(L)*B(L,3))
C
      BMATRX(3,1)=BMATRX(1,3)
C
      BMATRX(3,2)=BMATRX(2,3)
C
      BMATRX(3,3)=COEFFB    *(-R1(L)*(B(L,3)**2+B(L,1)*B(L,3)+A(L,3)**2+
     1A(L,1)*A(L,3))-R2(L)*(B(L,3)**2+B(L,2)*B(L,3)+A(L,3)**2+A(L,2)*
     2A(L,3))-R3(L)*(2.0*B(L,3)**2+2.0*A(L,3)**2)+2.0*AREA(L)*B(L,3))
C
C
C STORE ELEMENT MATRIX, AMATRX, INTO SYSTEM MATRIX, BIGA
C----------------------------------------------------------------------
C
      DC 20 K=1,3
      KK=ELNOD(L,K)
      DC 10 I=1,3
      II=ELNCD(L,I)
      DO 91 MX=1,LCON
      NCW =MX
      MM     =MNOD(KK,MX)
      IF (MM      .EQ. II) GO TO 92
   91 CONTINUE
   92 CCNTINUE
      BIGAB(KK,NOW)=BIGAB(KK,NOW)+VD(KK)*BMATRX(K,I)
   10 CCNTINUE
   20 CONTINUE
C
  200 CCNTINUE
C
      RETURN
C
      END
      SUBROUTINE YVET(NUMEL,NUMEQ,NUMSNP,LCON,IUPBP,PSIIV,NUPBP,BIGC,
     1    ITYPE,ELNOD,CM,BIGA,BIGAB,MNOD,BIGCC,PART,NWK,PSI,PW,DPSI,
     2   PDPSI,HOLD,DPT,PV,NCOUNT,ZOMEGA)
C
C
      INTEGER*4 ELNOD
C
      DIMENSION IUPBP(NUPBP),PSIIV(NUMSNP),BIGC(NUMSNP,LCON),
     1    ELNOD(NUMEL,3),CM(NUMEL,3,3,3),BIGA(NUMSNP,LCON),
     2    BIGAB(NUMSNP,LCON),MNOD(NUMSNP,LCON),BIGCC(NUMSNP,LCON),
     3    PART(NUMSNP,LCON),PDPSI(NUMSNP)
C
```

```
      DIMENSION  HOLD(NUMSNP),PSI(8,NUMSNP),DPT(NUMSNP),DPSI(NUMSNP),
     1   PW(NWK),PV(NUMSNP),ALFA(3,3)
C
      RETURN
C
C
C*****************************************************************************
C
             ENTRY YVETTE(PSI,T,M,DPSI,PW,IND)
C
C YVETTE IS AN EXTERNAL SUBROUTINE REQUIRED BY SUBROUTINE DVOGER
C
C
      KANT=8
C
C     SIZE IS THE CONVERGENT CRITERIA FOR SYSTEM SOLN.
      SIZE=.001
C
      DO 15 I=1,NUMSNP
      PV(I)=PSI(1,I)
      DPSI(I)=.0
   15 CONTINUE
      DO 50 I=1,NUPBP
      J=IUPBP(I)
      PV(J)=0.
   50 CONTINUE
      IF (NCOUNT .NE. 1) GO TO 05
C
      DO 27 MM=1,NUMSNP
      PSIIV(MM)=PV(MM)
      DO 28 I=1,NUMSNP
      BIGC(MM,I)=0.
   28 CONTINUE
   27 CONTINUE
      DO 210 L=1,NUMEL
      LL=ITYPE(L)
      IF(LL .NE. 0) GO TO 210
      N1=ELNOD(L,1)
      N2=ELNOD(L,2)
      N3=ELNOD(L,3)
      DO 86  J=1,3
      DO 90 N=1,3
      ALFA(N,J)=PSIIV(N1)*CM(L,N,J,1)+PSIIV(N2)*CM(L,N,J,2)
     1   +PSIIV(N3)*CM(L,N,J,3)
   90 CONTINUE
   86 CONTINUE
      DO 150 K=1,3
      KK=ELNOD(L,K)
```

```
      DO 280 I=1,3
      II=ELNOD(L,I)
C
      BIGC(KK,II)=BIGC(KK,II)+ALFA(K,I)*ZOMEGA(KK)
  280 CONTINUE
  150 CONTINUE
  210 CONTINUE
C
C        INSRRTION OF BOUNDARY POINTS
      DO 285 I=1,NUPBP
      II=IUPBP(I)
      DO 281 J=1,NUMSNP
      BIGC(J,II)=0.
      BIGC(II,J)=0.
  281 CONTINUE
  285 CONTINUE
      DO 311 I=1,NUMSNP
      DO 310 J=1,NUMSNP
      BIGCC(I,J)=0.
      PART(I,J)=.0
      DO 300 K=1,NUMSNP
      BIGCC(I,J)=BIGCC(I,J)+(BIGAB(K,J)- BIGC(K,J))*BIGA(I,K)
      PART(I,J)=PART(I,J)+(BIGAB(K,J)-2.*BIGC(K,J))*BIGA(I,K)
  300 CONTINUE
  310 CONTINUE
  311 CONTINUE
C
   05 CONTINUE
C
C
      IF (IND .EQ. 1) GO TO 200
C
      DO 81 K=1,NUMSNP
      DPT(K)=0.
C
      DO 80 I=1,LCON
      NAME=MNOD(K,I)
      DPT (K)=PV(NAME)*BIGCC(K,I)+DPT (K)
   80 CONTINUE
   81 CONTINUE
C
      IK= 0
      DO 120 IQ=1,NUMSNP
      DO 100 K=1,NUMEQ
      HOLD(K)=.0
      DPSI(K)=0.
C
C
```

```
      DC 82 I=2,LCON
      NAME=MNOD(K,I)
      HOLD(K)=PDPSI(NAME)*BIGA(K,I)+HOLD(K)
   82 CONTINUE
      DFSI(K)=(DPT (K)-HOLD(K))/BIGA(K,1)
C
      IF (DPSI(K) .EQ. 0.    .OR.
     1    ABS((PDPSI(K)-DPSI(K))/DPSI(K)) .LT. SIZE)
     2    IK=IK+1
      PDPSI(K)=DPSI(K)
  100 CONTINUE
      IF (IK .GE.  NUMEQ) GO TO 121
C
  120 CONTINUE
C
      WRITE(6,59)NUMSNP
   59 FORMAT (//2X,'CONVERGENCE NOT OBTAINED IN ',I4,' ITERATIONS.')
      GC TO 122
C
C
  121 CCNTINUE
      IF (IQ .GT. KANT)  WRITE(6,58) IQ
   58 FCRMAT (85X,'ITERATIONS REQUIRED FOR CONVERGENCE ',I4)
      IF(T.GT.0.0) RETURN
  122 CCNTINUE
      WRITE(6,60)T
   60 FORMAT(/2X,'FROM YVETTE',5X,'T=',G20.10)
      WRITE(6,65)(DPSI(K),K=1,NUMSNP)
      WRITE(6,65)(PSI(1,K),K=1,NUMSNP)
   65 FORMAT(10(1X,E11.5))
C
      RETURN
C
C
C
C     FOLLCWING IS ADDITIONAL REQD FOR MTH EQ 1
C
  200 CONTINUE
C     *                                                           *
C         AT PRESENT THERE EXIST BOTH CONCEPTUAL AND PROGRAMING
C         DIFFICULTIES IN COMPUTING THE JOCOBIAN PW.  WITHOUT
C         USING THE NXN MATRIX INVERSION TO CLEAR THE LHS.
C         EXAMINATION OF THIS PROBLEM TO INCLUDE POSSIBLE
C         CHANGES IN THE DVOGER ROUTINE ITSELF IS BEING
C         CONDUCTED BY OTHER INVESTIGATORS.
C
      RETURN
C
```

```
C   THIS  IS THE LAST OF THE EXTRA FOR MTH EQ 1
C
C
      END
      SUBROUTINE CRANKO (NUMEL,NUMSNP,NUPBP,LCON,H,TO,ERRVAL,
     1    EPSVAL,IUPBP,PSIIV,ITYPE,ELNOD,CM,MNOD,ZOMEGA,BIGAB,BIGA,
     2 IPT1,IPT2,IPT3,PV,PSTEP,BIGD,BIGE,ASTR,ESTR,
     3   BIGC,BIGCC,HOLD,TF)
C
C
C
      INTEGER*4 ELNOD
C
      DIMENSION IUPBP(NUPBP),PSIIV(NUMSNP),ITYPE(NUMSNP),
     1    ELNOD(NUMEL,3),CM(NUMEL,3,3,3),MNOD(NUMSNP,LCON),
     2    ZOMEGA(NUMSNP),BIGAB(NUMSNP,LCON),BIGA(NUMSNP,LCON),
     3    BATCH(400,4),PSTEP(NUMSNP),ALFA(3,3),PV(NUMSNP)
      DIMENSION BIGD(NUMSNP,LCON),BIGE(NUMSNP,LCON),
     1    ASTR(NUMSNP,LCON),ESTR(NUMSNP),BIGC(NUMSNP,LCON),
     2   BIGCC(NUMSNP,LCON),HOLD(NUMSNP)
C
C
C        DLT IS THE MAX TIME INTERVAL ATTEMPTED FOR ONE STEP.
C        EPSN IS THE CONVERGENCE CRITERA FOR  SOLN AT TIME T=T+DT
C
C
C        INITIALIZATION
C
      DLT=H
      PTIME=H
      T=TO
      LRGE = 1
      EPSN=ERRVAL
      STAR=EPSVAL
      NIT=0
      NUMEQ=NUMSNP-NUPBP
      MIN=NUMSNP/4
C
C
      NUF=NUMSNP/5
      XNP=NUMSNP
      XNUF=XNP/5.
      IF ((XNUF-FLOAT(NUF)) .LT. 1.)  NUF=NUF+1
C
      DO 05 I=1,NUMSNP
      PV(I)=.0
   05 CONTINUE
C
```

```
C
C        BUILD THE BIGCC MATRIX
C
  351 CONTINUE
C
      DO 07 I=1,NUPBP
      J=IUPBP(I)
      PSIIV(J)=0.
   07 CONTINUE
      DO 09 I=1,NUMSNP
      PSTEP(I)=PSIIV(I)
   09 CONTINUE
C
      DO 210 L=1,NUMEL
      LL=ITYPE(L)
      IF(LL .NE. 0) GO TO 210
      N1=ELNOD(L,1)
      N2=ELNOD(L,2)
      N3=ELNOD(L,3)
      DO 88 J=1,3
      DO 90 N=1,3
      ALFA(N,J)=PSIIV(N1)*CM(L,N,J,1)+PSIIV(N2)*CM(L,N,J,2)
     1   +PSIIV(N3)*CM(L,N,J,3)
   90 CONTINUE
   88 CONTINUE
      DO 150 K=1,3
      KK=ELNOD(L,K)
      DO 180 I=1,3
      II=ELNOD(L,I)
      DO 91 MX=1,LCON
      NOW =MX
      MM     =MNOD(KK,MX)
      IF (MM      .EQ. II) GO TO 92
   91 CONTINUE
   92 CONTINUE
      BIGC(KK,NOW)=BIGC(KK,NOW)+ALFA(K,I)*ZOMEGA(KK)
  180 CONTINUE
  150 CONTINUE
  210 CONTINUE
C     WRITE(6,755)
  755 FORMAT(///10X,' BIGC MATRIX BEFORE BOUNDRY POINTS'/)
C     WRITE(6,778)((BIGC(I,J),J=1,LCON) ,I=1,NUMSNP)
  778 FORMAT (2X,8(E14.5))
C
C          INSERTION OF BOUNDARY POINTS
      DO 185 I=1,NUPBP
      II=IUPBP(I)
      DO 191 MX=1,LCON
```

```
      BIGC(II,MX)=.0
  191 CONTINUE
  185 CCNTINUE
C     WRITE(6,757)
  757 FORMAT(///10X,'BIGC MATRIX WITH BOUNDARY POINTS'/)
C     WRITE(6,778)((BIGC(I,J),J=1,LCON) ,I=1,NUMSNP)
C
C
      DC 310  I=1,NUMSNP
      DO 310  J=1,LCON
      BIGCC(I,J)=BIGAB(I,J)-BIGC(I,J)
  310 CONTINUE
C     WRITE(6,759)
  759 FORMAT (///10X,'BIGCC MATRIX'/)
C     WRITE(6,778) ((BIGCC(I,J),J=1,LCON),I=1,NUMSNP)
C
C
C
C        BEGIN STEPING OUT IN TIME.
C
      STP = 4.
      ITRY=0
C
C        COMPUTE PSI BASED ON A TRIAL DT.
      DO 55 JC=1,12
C
C        FIRST COMPUTE (2/DT)*A=ASTR, ASTR-C=D, ASTR+D=E
      DS=2./DLT
      DO 15 KA=1,NUMSNP
      DO 13 KB=1,LCON
      ASTR(KA,KB)=DS*BIGA(KA,KB)
      BIGD(KA,KB)=ASTR(KA,KB)-BIGCC(KA,KB)
      BIGE(KA,KB)=ASTR(KA,KB)+BIGCC(KA,KB)
   13 CONTINUE
   15 CONTINUE
C
      DO 19 KA=1,NUMSNP
      ESTR(KA)=0.
      DO 17 KB=1,LCON
      NAME = MNOD(KA,KB)
      ESTR(KA)=ESTR(KA)+BIGE(KA,KB)*PSIIV(NAME)
   17 CONTINUE
   19 CONTINUE
C
C        THE SYSTEM IS NOW IS THE FAMILIAR FORM D(I,J)*PV(J)=ESTR(J)
C        AND MAY BE SOLVED USING ANY TECHNIQUE FOR LINEAR SIMULTANEOUS
C        EQNS.  HERE GAUSS-SEIDEL ITERATION IS USED.
C
C        ACCOMPLISH THE FIRST ITERATION BASED ON THE PV VALUE AT THE
```

```
C         LAST SUCCESSFUL TIME POINT.  SECOND AND SUCCESSIVE ITERATIONS
C         ARE BASED ON UPDATED VALUED OF PV.  A TEST FOR CONVERGENCE IS
C         MADE, AND FAILURE RESULTS IN ANOTHER ATTEMPT WITH A SMALLER DT
C
C         CREATION OF ESTR(I)=BIGE(I,J)*PSIIV(I)
C         PERFORMED ONCE ON EACH ATTEMPT TO FIND A PROPER TIME INTERVAL.
C
      DO 45 MTRY=1,NUMEQ
      KT=0
      DO 25 K=1,NUMEQ
      HOLD(K)=0.
      PV(K)=0.
      DO 20 L=2,LCON
      NAME = MNOD(K,L)
      HOLD(K)= HOLD(K) + BIGD(K,L)*PSTEP(NAME)
   20 CONTINUE
      PV(K)=(ESTR(K)-HOLD(K))/BIGD(K,1)
      PDIF=(PV(K)-PSTEP(K))/PV(K)
   24 CONTINUE
      IF (ABS(PDIF) .LT. EPSN) KT =KT+1
      PSTEP(K)=PV(K)
   25 CONTINUE
      IF (KT .GE. NUMEQ)  GO TO 65
   45 CONTINUE
C
C
  291 FORMAT (//10X,'NO SOLN FOUND IN INTERVAL ',G10.4,'  PLUS ',
     1   G10.4)
      WRITE(6,291) T,DLT
C
C         PENALTY --- MAKE THE INTERVAL EVEN SMALLER.
      DLT=DLT/STP
      STP=STP+2.
      ITRY=ITRY+1
   55 CONTINUE
C
C
      WRITE(6,292)
  292 FORMAT(/10X,'A SOLN IS NOT POSSIBLE.  THE PROGRAM SURRENDERS.')
C
      STOP
C
C
   65 CONTINUE
      T=T + DLT
      ITRA=MTRY+NUMEQ*ITRY
      WRITE(6,350) T,DLT,ITRA
  350 FORMAT   (5X,'T= ',G12.6,4X,'DT= ',G12.6,4X,
```

```
     1   'ITERATIONS REQUIRED = ',I3)
C
C        IF THE SCLN REQUIRED LESS ITERATIONS THAN PREVIOUSLY,
C        INCREASE THE SIZE OF THE INITIAL TIME STEP.
      IF (ITRA .LT. LRGE .OR. ITRA .LT. MIN) CLT=CLT*1.5
      LRGE = ITRA
C
      IF ((T-PTIME)/PTIME .LT. STAR)  GO TO 377
      PTIME=T
      WRITE (6,354)
  354 FORMAT(/(5(4X,'NODE',7X,'PSI',4X)))
      DO 355 I=1,NUF
      I1=I+NUF
      I2=I+2*NUF
      I3=I+3*NUF
      I4=I+4*NUF
      WRITE(6,356) (I,PV(I),I1,PV(I1),I2,PV(I2),I3,PV(I3),I4,PV(I4))
  355 CCNTINUE
  356 FORMAT(5(4X,I3,3X,1PE12.4))
      WRITE(6,357)
  357 FORMAT(/10X)
  700 FORMAT (//1X,'VALUES OF IPT-1, IPT-2, IPT-3'/)
C
      NIT= 1 + NIT
      IF (NIT .GT. 400)  GO TO 377
      BATCH (NIT,1)=T
      BATCH (NIT,2)= PV(IPT1)
      BATCH (NIT,3)= PV(IPT2)
      BATCH (NIT,4)= PV(IPT3)
  377 CCNTINUE
C
      DO 27 MN=1,NUMSNP
      PSIIV(MN)=PV(MN)
      DO 28 I=1,LCON
      BIGC(MN,I)=.0
   28 CCNTINUE
   27 CONTINUE
      IF (T .LT. TF) GO TO 351
  399 FORMAT (10X,I4)
      WRITE (6,399) NIT
      WRITE (3) BATCH
      WRITE (4) BATCH
      STCP
      END
```

```
C      *              PROGRAM FED-2                                  *
C      *              FINITE ELEMENT SOLUTION OF NONLINEAR           *
C      *          REACTOR DYNAMICS IN TWO-DIMENTIONAL SPACE          *
C      ***** LINEARIZED VERSION**********
C
C
       INTEGER*4 SYSNOD,ELEMNT,ELNOD
C
       DIMENSION    PART(NUMSNP,NUMSNP),BIGA(NUMSNP,NUMSNP),
      1   BIGAB(NUMSNP,NUMSNP),BIGC(NUMSNP,NUMSNP),
      2   BIGCC(NUMSNP,NUMSNP),SYSNOD(NUMSNP),ELEMNT(NUMEL),
      3 ELNOD(NUMEL,3),IUPBP(NUPBP),ITYPE(NUMEL),V(NUMSNP),VD(NUMSNP),
      4   D(NUMSNP),SGA(NUMSNP),SGF(NUMSNP),ALPHA(NUMSNP),
      5   ZLMBDA(NUMSNP),VLMBDA(NUMSNP),ZOMEGA(NUMSNP),RHO(NUMSNP),
      6   R(NUMSNP),Z(NUMSNP),R1(NUMEL),Z1(NUMEL),R2(NUMEL),Z2(NUMEL),
      7   R3(NUMEL),Z3(NUMEL),A(NUMEL,3),B(NUMEL,3),YMAX(NUMSNP),
      8   ERROR(NUMSNP),AREA(NUMEL),CM(NUMEL,3,3,3),PSIIV(NUMSNP),
      9    PDPSI(NUMSNP),DPSI(NUMSNP),PW(NWK),PSI(8,NUMSNP)
C
       DIMENSICN BATCH(400,4),TITLE(20),ALFA(3,3)
C
C
       DC 05 I=1,NUMSNP
       ERROR(I)=0.
        PDPSI(I)=0.
       YMAX(I)=1.
    05 CONTINUE
C
       NUF=NUMSNP/5
       XNP=NUMSNP
       XNUF=XNP/5.
       IF ((XNUF-FLOAT(NUF)) .LT. 1.)  NUF=NUF+1
C
C
C
C******************************************************************@
C
C
C  GENERAL PROBLEM DATA
C******************************************************************@
C
       READ(5,12) (IUPBP(I),I=1,NUPBP)
       READ(5,10) MTH,MAXDER,NCOUNT
    10 FCRMAT(8I10)
    12 FCRMAT(16I5)
C
       WRITE(6,11) NUMEL,NUPBP,NUMSNP,NFULEL
    11 FORMAT(//2X,'# OF ELEMENTS=',I5//2X,'# OF UPPER BDRY NODAL POINTS=
```

```
     1',I5//2X,'# OF SYSTEM NODAL POINTS=',I5//2X,'# OF FUEL ELEMENTS=',
     2I5)
C
      READ(5,15) ZNU,FISFAC,HBAR,EPSVAL,ERRVAL,AFUEL
      READ(5,19)TO,H,TF,HMIN,HMAX
      READ(5,15) (V(I),I=1,NUMSNP)
      READ(5,15) (D(I),I=1,NUMSNP)
      READ(5,15) (SGA(I),I=1,NUMSNP)
      READ(5,15) (SGF(I),I=1,NUMSNP)
      READ(5,15) (ALPHA(I),I=1,NUMSNP)
      READ(5,18) (PSIIV(I),I=1,NUMSNP)
   15 FORMAT(8G10.5)
C
C     CALCULATE PHYSICAL CONSTANTS FROM NUCLEAR DATA
C
      PI= 3.1415927
C
      DC 25 I=1,NUMSNP
      VC(I)=V(I)*D(I)
      ZLMBDA(I)=ZNU*SGF(I)/SGA(I)-1.
      VLMBCA(I)=V(I)*ZLMBDA(I)*SGA(I)
      ZCMEGA(I)=V(I)*SGA(I)*ALPHA(I)*FISFAC*SGF(I)/HBAR
      RHO(I)=ZNU*(SGF(I)-0.00621835)/(SGA(I)*0.003297)
      IF (I .GT. NFULEL)  RHO(I)=.0
   25 CCNTINUE
C
C
   18 FCRMAT(5E15.4)
   19 FORMAT(2E20.4,4G10.5)
  998 FCRMAT(20A4)
C
C
C--------- GEOMETRY OF SYSTEM NODAL POINTS ------------------------------------
C
      WRITE(6,40)
   40 FORMAT   (///1X,'GEOMETRY OF SYSTEM NODAL PCINTS'///)
C
      READ(5,41) (SYSNOD(I),R(I),Z(I),I=1,NUMSNP)
   41 FORMAT( I10,2G15.8)
C
C
      WRITE      (6,60) (SYSNOD(I),R(I),Z(I),I=1,NUMSNP)
   60 FORMAT (1X,'SYSTEM NODAL POINT NO. ',I3,5X,'R COORD.= ',F10.6,5X,'
     1Z COORD.= ',F10.6)
C
C*****************************************************************************
C CORRESPONDENCE TABLE BETWEEN SYSTEM AND ELEMENT NODAL POINTS
C*****************************************************************************@
```

```
C
C
      WRITE(6,73)
   73 FORMAT(//2X,'CONNECTIVITY MATRIX',//,' EL #',33X,'TYPE'//)
      DO 75 I=1,NUMEL
      READ(5,76) ELEMNT(I),ELNOD(I,1),ELNOD(I,2),ELNOD(I,3),ITYPE(I)
      WRITE(6,77) ELEMNT(I),ELNOD(I,1),ELNOD(I,2),ELNOD(I,3),ITYPE(I)
   75 CONTINUE
   76 FORMAT (5I10)
   77 FORMAT (2X,I3,4I10)
C
C
C
C*****************************************************************************@
C CALCULATE GEOMETRY FOR EACH ELEMENT
C*****************************************************************************@
C
      WRITE(6,110)
  110 FORMAT (1X///1X,'GEOMETRY CALCULATIONS FOR EACH ELEMENT')
C
C-----------------------------------------------------------------------------
C LOAD SYSTEM NODE COORDINATES (R,Z) INTO
C ELEMENT NODE COORDINATES (R1,R2,R3,Z1,Z2,Z3)
C-----------------------------------------------------------------------------
C
C
      DO 100 I=1,NUMEL
C
      J=ELNOD(I,1)
      R1(I)=R(J)
      Z1(I)=Z(J)
C
      K=ELNOD(I,2)
      R2(I)=R(K)
      Z2(I)=Z(K)
C
      L=ELNOD(I,3)
      R3(I)=R(L)
      Z3(I)=Z(L)
C
C
      WRITE      (6,105) I,R1(I),Z1(I),R2(I),Z2(I),R3(I),Z3(I)
  100 CONTINUE
  105 FORMAT (3X,I3,3X,   7(F10.6,1X))
C
C-----------------------------------------------------------------------------
C COMPUTE A1,A2,A3,B1,B2,B3,AREA FOR EACH ELEMENT
C-----------------------------------------------------------------------------
```

```
C
      WRITE       (6,120)
  120 FORMAT (1X///1X,'ELEMENT',5X,'A1',9X,'A2',9X,'A3',9X,'B1',9X,'B2',
     19X,'B3',8X,'AREA'/)
C
      DO 140 I=1,NUMEL
      A(I,1)=R3(I)-R2(I)
      A(I,2)=R1(I)-R3(I)
      A(I,3)=R2(I)-R1(I)
      B(I,1)=Z2(I)-Z3(I)
      B(I,2)=Z3(I)-Z1(I)
      B(I,3)=Z1(I)-Z2(I)
      AREA(I)=0.5*(A(I,2)*B(I,1)-A(I,1)*B(I,2))
C
      WRITE(6,130) I,A(I,1),A(I,2),A(I,3),B(I,1),B(I,2),B(I,3),
     1AREA(I)
  140 CONTINUE
  130 FORMAT (3X,I3,3X,   7(F12.7,1X))
C
C
C-----------------------------------------------------------------------
C
C
C--------- ZERO THE BIGA,BIGB,BIGC AND BIGAB MATRICES--------------------------
C
C
C
      DO 194  KK=1,NUMSNP
      DO 193  II=1,NUMSNP
      BIGA(KK,II)=0.
      PART(KK,II)=.0
      BIGAB(KK,II)=0.0
      BIGC(KK,II)=0.
  193 CONTINUE
  194 CONTINUE
C
C
C-----------------------------------------------------------------------
C     PEPSI CALCULATES THE BIGA MATRIX AND PART OF THE BIGAB
C-----------------------------------------------------------------------
C
      CALL PEPSI (NUMEL,NUMSNP,R1,R2,R3,AREA,ELNOD,
     1    BIGA,BIGAB,VLMBDA)
C
      WRITE(6,195)
  195 FORMAT(//5X,'BIGA MATRIX',//)
  196 FORMAT(//5X,'BIGAB MATRIX FROM PEPSI, NO DEL**2 CONTRIBUTION',//)
C
```

```
C
C-----------------------------------------------------------------------
C     CRUISE CALCULATES THE BIGAB MATRIX
C-----------------------------------------------------------------------
C
      CALL CRUISE (NUMEL,NUMSNP,      R1,R2,R3,AREA,ELNOD,
     1    BIGAB,     A,B,VD)
C
      WRITE(6,197)
  197 FORMAT(//5X,'BIGAB MATRIX FROM CRUISE, WITH DEL**2 TERM',//)
C
      DO 255 I=1,NUPBP
      II=IUPBP(I)
      DO 251 J=1,NUMSNP
      BIGA(J,II)=0.
      BIGA(II,J)=0.
      BIGAB(J,II)=0.
      BIGAB(II,J)=0.
  251 CONTINUE
      BIGA(II,II)=1.
      BIGAB(II,II)=0.
  255 CONTINUE
C
C
      D1=1.
      CALL LINV3F(BIGA,B,1,NUMSNP,NUMSNP,D1,D2,PART,IER)
C
      WRITE(6,250)
  250 FORMAT(//5X,'BIGA INVERSE',//)
C
C
C     WRITE OUT NUCLEAR DATA
C
      WRITE(6,3105) ZNU,FISFAC,HBAR,EPSVAL,ERRVAL,AFUEL
 3105 FORMAT(//5X,'NUCLEAR DATA'/2X,'ZNU=',G12.6/2X,'FISFAC=',G12.6/2X,
     1 'HBAR=',G12.6/2X,'EPSVAL=',G12.6/2X,'ERRVAL=',G12.6/2X,'AFUEL=',
     2 G12.6)
      WRITE(6,3110)
 3110 FORMAT(1H0,2X,'NODE',6X,'D',8X,'SGA',9X,'SGF',8X,'ALPHA',9X,'VD',
     1 7X,'ZLMBDA',7X,'VLMBDA',7X,'ZOMEGA',8X,'V',9X,'RHO($)')
      DO 3115 I=1,NUMSNP
      WRITE(6,3120) (I,D(I),SGA(I),SGF(I),ALPHA(I),VD(I),ZLMBDA(I),
     1 VLMBDA(I),ZOMEGA(I),V(I),RHO(I))
 3115 CONTINUE
 3120 FORMAT(2X,I4,10(1PE12.4))
C
C     INTEGRATION BY DVOGER
C
```

```
      EXTERNAL YVETTE
            CALL YVET(NUMEL,NUMEQ,NUMSNP,        IUPBP,PSIIV,NUPBP,BIGC,
     1    ITYPE,ELNOD,CM,BIGA,BIGAB,        BIGCC,PART,NWK,PSI,PW,DPSI,
     2   PDPSI,HOLD,DPT,PV,NCOUNT,ZOMEGA,IPT1,IPT2,IPT3)
C
C--------- INITIALIZE ARGUMENTS OF DVOGER ---------------------------------------
C
      PTIME= 1E-17
      STAR=.01
      NIT=0
      T=TO
      JSTART=0
      EPS= EPSVAL
      IER=0
C
      WRITE(6,999) TITLE
  999 FORMAT(1H1,20A4)
C
      WRITE(6,317)
      WRITE(6,318) T,MTH,MAXDER,JSTART,H,HMIN,HMAX,EPS,NCOUNT
  317 FORMAT  (//10X,'INITIAL ARGUMENTS',//)
  318 FORMAT(2X,'T=',G10.4,2X,'MTH=',I4,2X,'MAXDER=',I4,2X,'JSTART=',I4,
     12X,'H=',G10.4,2X,'HMIN=',G10.4,2X,'HMAX=',G10.4,2X,'EPS=',G10.4,2X
     2,'NCOUNT=',I2,/)
C
C
      DO 320 I=1,NUMSNP
      PSI(1,I)=PSIIV(I)
  320 CONTINUE
C
      DO 321 I=1,NUPBP
      J=IUPBP(I)
      PSI(1,J)=0.0
  321 CONTINUE
C
      NUMEQ=NUMSNP-NUPBP
C
C
C
C     ************************************************************
C     CONSTRUCT THE BIGC MATRIX ON THE ELEMENT LEVEL
C-------------------------------------------------------------------------------
C
C
      DO 200 L=1,NUMEL
C
      DO 333 I=1,3
      DO 332 J=1,3
```

```
      DO 331 K=1,3
      CM(L,I,J,K)=0.
  331 CONTINUE
  332 CONTINUE
  333 CONTINUE
C
      LL=ITYPE(L)
      IF(LL.NE.0) GO TO 200
C
      CC=PI*AREA(L)/180.
C
      CM(L,3,3,3)=CC*(6.0*R1(L)+6.0*R2(L)+24.*R3(L))
      CM(L,3,3,2)=CC*(2.0*R1(L)+4.0*R2(L)+6.0*R3(L))
      CM(L,3,3,1)=CC*(4.0*R1(L)+2.0*R2(L)+6.0*R3(L))
      CM(L,3,2,3)=CC*(2.0*R1(L)+4.0*R2(L)+6.0*R3(L))
      CM(L,3,2,2)=CC*(2.0*R1(L)+6.0*R2(L)+4.0*R3(L))
      CM(L,3,2,1)=CC*(2.0*R1(L)+2.0*R2(L)+2.0*R3(L))
      CM(L,3,1,3)=CC*(4.0*R1(L)+2.0*R2(L)+6.0*R3(L))
      CM(L,3,1,2)=CC*(2.0*R1(L)+2.0*R2(L)+2.0*R3(L))
      CM(L,3,1,1)=CC*(6.0*R1(L)+2.0*R2(L)+4.0*R3(L))
      CM(L,2,3,3)=CC*(2.0*R1(L)+4.0*R2(L)+6.0*R3(L))
      CM(L,2,3,2)=CC*(2.0*R1(L)+6.0*R2(L)+4.0*R3(L))
      CM(L,2,3,1)=CC*(2.0*R1(L)+2.0*R2(L)+2.0*R3(L))
      CM(L,2,2,3)=CC*(2.0*R1(L)+6.0*R2(L)+4.0*R3(L))
      CM(L,2,2,2)=CC*(6.0*R1(L)+24.*R2(L)+6.0*R3(L))
      CM(L,2,2,1)=CC*(4.0*R1(L)+6.0*R2(L)+2.0*R3(L))
      CM(L,2,1,3)=CC*(2.0*R1(L)+2.0*R2(L)+2.0*R3(L))
      CM(L,2,1,2)=CC*(4.0*R1(L)+6.0*R2(L)+2.0*R3(L))
      CM(L,2,1,1)=CC*(6.0*R1(L)+4.0*R2(L)+2.0*R3(L))
      CM(L,1,3,3)=CC*(4.0*R1(L)+2.0*R2(L)+6.0*R3(L))
      CM(L,1,3,2)=CC*(2.0*R1(L)+2.0*R2(L)+2.0*R3(L))
      CM(L,1,3,1)=CC*(6.0*R1(L)+2.0*R2(L)+4.0*R3(L))
      CM(L,1,2,3)=CC*(2.0*R1(L)+2.0*R2(L)+2.0*R3(L))
      CM(L,1,2,2)=CC*(4.0*R1(L)+6.0*R2(L)+2.0*R3(L))
      CM(L,1,2,1)=CC*(6.0*R1(L)+4.0*R2(L)+2.0*R3(L))
      CM(L,1,1,3)=CC*(6.0*R1(L)+2.0*R2(L)+4.0*R3(L))
      CM(L,1,1,2)=CC*(6.0*R1(L)+4.0*R2(L)+2.0*R3(L))
      CM(L,1,1,1)=CC*(24.*R1(L)+6.0*R2(L)+6.0*R3(L))
C
  200 CCNTINUE
C
  351 CONTINUE
      IF (NCOUNT .EQ. 1)GO TO 312
C
      DO 210 L=1,NUMEL
      LL=ITYPE(L)
      IF(LL .NE. 0) GO TO 210
      N1=ELNOD(L,1)
```

```
      N2=ELNOD(L,2)
      N3=ELNOD(L,3)
      DO 88 J=1,3
      DO 90 N=1,3
      ALFA(N,J)=PSIIV(N1)*CM(L,N,J,1)+PSIIV(N2)*CM(L,N,J,2)
     1   +PSIIV(N3)*CM(L,N,J,3)
   90 CONTINUE
   88 CONTINUE
      DO 150 K=1,3
      KK=ELNOD(L,K)
      DO 280 I=1,3
      II=ELNOD(L,I)
C
      BIGC(KK,II)=BIGC(KK,II)+ALFA(K,I)*ZOMEGA(KK)
C
  280 CONTINUE
  150 CONTINUE
  210 CONTINUE
C
C         INSRRTION OF BOUNDARY POINTS
      DO 285 I=1,NUPBP
      II=IUPBP(I)
      DO 281 J=1,NUMSNP
      BIGC(J,II)=0.
      BIGC(II,J)=0.
  281 CONTINUE
  285 CONTINUE
      DO 310 I=1,NUMSNP
      DO 310 J=1,NUMSNP
      BIGCC(I,J)=0.
      PART(I,J)=.0
      DO 300 K=1,NUMSNP
      BIGCC(I,J)=BIGCC(I,J)+(BIGAB(K,J)- BIGC(K,J))*BIGA(I,K)
      PART(I,J)=PART(I,J)+(BIGAB(K,J)-2.*BIGC(K,J))*BIGA(I,K)
  300 CONTINUE
  310 CONTINUE
C
  312 CONTINUE
C
C
C         ***********************************************************
C         * SOLVE NON-LINEAR FLUX EQUATIONS WITH SUBROUTINE DVOGER *
C         ***********************************************************
C
  350 CALL DVOGER(YVETTE,PSI,T,NUMSNP,MTH,MAXDER,JSTART,H,HMIN,HMAX,EPS,
     1YMAX,ERROR,PW,IER)
C
C
```

```
      IF(IER.EQ.0) GO TO 3540
      JSTART=-1
      H=HMIN*.1
      HMIN=H*.1
      GO TO 350
 3540 CONTINUE
      JSTART=1
      IF((T-PTIME)/PTIME .LT. STAR)  GO TO 362
      PTIME=T
      WRITE(6,3510)T,H,JSTART,IER
 3510 FORMAT(//2X,'T=',G12.6,2X,'H=',G12.6,2X,'JSTART=',I4,2X,'IER=',I4)
C
      WRITE(6,354)
  354 FORMAT(/(5(4X,'NODE',7X,'PSI',4X)))
      DO 355 I=1,NUF
      I1=I+NUF
      I2=I+2*NUF
      I3=I+3*NUF
      I4=I+4*NUF
      WRITE(6,356) (I,PSI(1,I),I1,PSI(1,I1),I2,PSI(1,I2),I3,PSI(1,I3),I4
     1,PSI(1,I4))
  355 CONTINUE
  356 FORMAT(5(4X,I3,3X,1PE12.4))
C
      NIT= 1 + NIT
      IF (NIT .GT. 400)  GO TO 377
      BATCH (NIT,1)=T
      BATCH (NIT,2)=PSI(1,IPT1)
      BATCH (NIT,3)=PSI(1,IPT2)
      BATCH (NIT,4)=PSI(1,IPT3)
  377 CONTINUE
C
C
      DO 360 I=1,NUMSNP
        R(I)=PSI(2,I)/H
  360 CONTINUE
C
      WRITE(6,357)
  357 FORMAT(/(5(4X,'NODE',7X,'DPSI',3X)))
      DO 359 I=1,NUF
      I1=I+NUF
      I2=I+2*NUF
      I3=I+3*NUF
      I4=I+4*NUF
      WRITE(6,356) (I,R(I),I1,R(I1),I2,R(I2),I3,R(I3),I4,R(I4))
  359 CONTINUE
  362 CONTINUE
C
```

```
C
C
      DO 27 MM=1,NUMSNP
      PSIIV(MM)=PSI(1,MM)
      DC 28 I=1,NUMSNP
      BIGC(MM,I)=0.
   28 CONTINUE
   27 CONTINUE
      IF (T .LT. TF) GO TO 351
C
C
  399 FCRMAT (10X,I4)
      WRITE (6,399) NIT
      WRITE (3) BATCH
      WRITE (4) BATCH
C
      STOP
      END
      SUBROUTINE PEPSI (NUMEL,NUMSNP,R1,R2,R3,AREA,ELNOD,
     1   BIGA,BIGAB,VLMBDA)
C
C
C
C************************************************************************
C PEPSI CALCULATES THE D-MATRIX MULTIPLYING THE D(CY)/DT TERM
C************************************************************************
C
      INTEGER*4 ELNOD
      DIMENSICN  R1(NUMEL),R2(NUMEL),R3(NUMEL),BIGA(NUMSNP,NUMSNP),
     1    BIGAB(NUMSNP,NUMSNP),                   AMATRX(3,3),
     2     VLMBDA(NUMSNP),AREA(NUMEL),ELNOD(NUMEL,3)
      PI=3.1415927
C
C
C
C
C------------------------------------------------------------------------
C CALCULATE THE 3X3 D(I,J) MATRIX FOR ELEMENT L
C------------------------------------------------------------------------
C
C
      DO 200 L=1,NUMEL
      COEFFA= (PI/30.0) * AREA(L)
C
C
      AMATRX(1,1)=COEFFA   *(6.0*R1(L)+2.0*R2(L)+2.0*R3(L))
      AMATRX(1,2)=COEFFA   *(2.0*R1(L)+2.0*R2(L)+R3(L))
      AMATRX(1,3)=COEFFA   *(2.0*R1(L)+R2(L)+2.0*R3(L))
```

```
      AMATRX(2,1)=AMATRX(1,2)
      AMATRX(2,2)=COEFFA    *(2.0*R1(L)+6.0*R2(L)+2.0*R3(L))
      AMATRX(2,3)=COEFFA    *(R1(L)+2.0*R2(L)+2.0*R3(L))
      AMATRX(3,1)=AMATRX(1,3)
      AMATRX(3,2)=AMATRX(2,3)
      AMATRX(3,3)=COEFFA    *(2.0*R1(L)+2.0*R2(L)+6.0*R3(L))
C
C-----------------------------------------------------------------------
C STORE ELEMENT MATRIX, AMATRX, INTO SYSTEM MATRIX, BIGA
C-----------------------------------------------------------------------
C
      DO 20 K=1,3
      KK=ELNOD(L,K)
      DO 10 I=1,3
      II=ELNOD(L,I)
      BIGA(KK,II)=BIGA(KK,II)+AMATRX(K,I)
      BIGAB(KK,II)=BIGAB(KK,II)+VLMBDA(KK)*AMATRX(K,I)
   10 CONTINUE
   20 CONTINUE
C
  200 CONTINUE
C
      RETURN
      END
      SUBROUTINE CRUISE (NUMEL,NUMSNP,      R1,R2,R3,AREA,ELNOD,BIGAB,
     1          A,B,VD)
C
C
C     CRUISE CALCULATES THE BIGAB MATRIX
C***********************************************************************
C
C
C
C
      INTEGER*4 ELNOD
C
      DIMENSION R1(NUMEL),R2(NUMEL),R3(NUMEL),AREA(NUMEL),
     1                        BIGAB(NUMSNP,NUMSNP),BMATRX(3,3),
     2   ELNOD(NUMEL,3),A(NUMEL,3),B(NUMEL,3),VD(NUMSNP)
C
      PI=3.1415927
C
C CALCULATE THE 3X3 B(I,J) MATRIX FOR ELEMENT L
C-----------------------------------------------------------------------
C
C
      DO 200 L=1,NUMEL
C
```

```
      COEFFB= PI / (6.0 * AREA(L) )
C
      BMATRX(1,1)=COEFFB    *(-R1(L)*(2.0*B(L,1)**2+2.0*A(L,1)**2)-R2(L)*
     1(B(L,1)*B(L,2)+B(L,1)**2+A(L,1)*A(L,2)+A(L,1)**2)-R3(L)*(B(L,1)*
     2B(L,3)+B(L,1)**2+A(L,1)*A(L,3)+A(L,1)**2)+2.0*AREA(L)*B(L,1))
C
      BMATRX(1,2)=COEFFB    *(-R1(L)*(2.0*B(L,1)*B(L,2)+2.0*A(L,1)*
     1A(L,2))-R2(L)*(B(L,2)**2+B(L,1)*B(L,2)+A(L,2)**2+A(L,1)*A(L,2))-
     2R3(L)*(B(L,2)*B(L,3)+B(L,1)*B(L,2)+A(L,2)*A(L,3)+A(L,1)*A(L,2))+
     32.0*AREA(L)*B(L,2))
C
      BMATRX(1,3)=COEFFB    *(-R1(L)*(2.0*B(L,1)*B(L,3)+2.0*A(L,1)*
     1A(L,3))-R2(L)*(B(L,2)*B(L,3)+B(L,1)*B(L,3)+A(L,2)*A(L,3)+A(L,1)*
     2A(L,3))-R3(L)*(B(L,3)**2+B(L,1)*B(L,3)+A(L,3)**2+A(L,1)*A(L,3))+
     32.0*AREA(L)*B(L,3))
C
      BMATRX(2,1)=BMATRX(1,2)
C
      BMATRX(2,2)=COEFFB    *(-R1(L)*(B(L,2)**2+B(L,1)*B(L,2)+A(L,2)**2+
     1A(L,1)*A(L,2))-R2(L)*(2.0*B(L,2)**2+2.0*A(L,2)**2)-R3(L)*(B(L,3)*
     2B(L,2)+B(L,2)**2+A(L,3)*A(L,2)+A(L,2)**2)+2.0*AREA(L)*B(L,2))
C
      BMATRX(2,3)=COEFFB    *(-R1(L)*(B(L,2)*B(L,3)+B(L,1)*B(L,3)+A(L,2)*
     1A(L,3)+A(L,1)*A(L,3))-R2(L)*(2.0*B(L,2)*B(L,3)+2.0*A(L,2)*A(L,3))-
     2R3(L)*(B(L,3)**2+B(L,2)*B(L,3)+A(L,3)**2+A(L,2)*A(L,3))+
     32.0*AREA(L)*B(L,3))
C
      BMATRX(3,1)=BMATRX(1,3)
C
      BMATRX(3,2)=BMATRX(2,3)
C
      BMATRX(3,3)=COEFFB    *(-R1(L)*(B(L,3)**2+B(L,1)*B(L,3)+A(L,3)**2+
     1A(L,1)*A(L,3))-R2(L)*(B(L,3)**2+B(L,2)*B(L,3)+A(L,3)**2+A(L,2)*
     2A(L,3))-R3(L)*(2.0*B(L,3)**2+2.0*A(L,3)**2)+2.0*AREA(L)*B(L,3))
C
C
C
C STORE ELEMENT MATRIX, AMATRX, INTO SYSTEM MATRIX, BIGA
C-------------------------------------------------------------------------
      DC 20 K=1,3
      KK=ELNOD(L,K)
      DO 10 I=1,3
      II=ELNOD(L,I)
C
      BIGAB(KK,II)=BIGAB(KK,II)+VD(KK)*BMATRX(K,I)
C
   10 CONTINUE
   20 CCNTINUE
```

```
C
  200 CONTINUE
C
      RETURN
C
      END
      SUBROUTINE YVET( NUMEL,NUMEQ,NUMSNP,      IUPBP,PSIIV,NUPBP,BIGC,
     1     ITYPE,ELNOD,CM,BIGA,BIGAB,      BIGCC,PART,NWK,PSI,PW,DPSI,
     2    PDPSI,HOLD,DPT,PV,NCOUNT,ZOMEGA,IPT1, IPT2, IPT3)
C
C
C*************************************************************************
C YVETTE IS AN EXTERNAL SUBROUTINE REQUIRED BY SUBROUTINE CVOGER
C*************************************************************************
C
C
      INTEGER*4 ELNOD
C
      DIMENSION IUPBP(NUPBP),PSIIV(NUMSNP),BIGC(NUMSNP,NUMSNP),
     1     ELNOD(NUMEL,3),CM(NUMEL,3,3,3),BIGA(NUMSNP,NUMSNP),
     2     BIGAB(NUMSNP,NUMSNP), BIGCC(NUMSNP,NUMSNP),
     3     PART(NUMSNP,NUMSNP),PDPSI(NUMSNP),PV(NUMSNP),PSI(8,NUMSNP),
     4    ALFA(3,3),DPSI(NUMSNP),PW(NWK),ITYPE(NUMEL)
C
      PI=3.1415927
C
   55 FORMAT (2(3X,1PE12.4))
      WRITE(6,55) BIGA(1,1),BIGAB(1,1)
      RETURN
C
C
           ENTRY YVETTE(PSI,T,M,DPSI,PW,IND)
C
C
      DO 15 I=1,NUMSNP
      PV(I)=PSI(1,I)
   15 CONTINUE
      DO 50 I=1,NUPBP
      J=IUPBP(I)
      PV(J)=0.
   50 CONTINUE
C
      IF (NCOUNT .NE. 1) GO TO 05
C
      DO 27 MM=1,NUMSNP
      PSIIV(MM)=PV(MM)
      DO 28 I=1,NUMSNP
      BIGC(MM,I)=0.
```

```
   28 CONTINUE
   27 CONTINUE
      DO 210 L=1,NUMEL
      LL=ITYPE(L)
      IF(LL .NE. 0) GO TO 210
      N1=ELNOD(L,1)
      N2=ELNOD(L,2)
      N3=ELNOD(L,3)
      DO 86  J=1,3
      DC 90 N=1,3
      ALFA(N,J)=PSIIV(N1)*CM(L,N,J,1)+PSIIV(N2)*CM(L,N,J,2)
     1   +PSIIV(N3)*CM(L,N,J,3)
   90 CCNTINUE
   86 CONTINUE
      DO 150 K=1,3
      KK=ELNOD(L,K)
      DO 280 I=1,3
      II=ELNOD(L,I)
C
      BIGC(KK,II)=BIGC(KK,II)+ALFA(K,I)*ZOMEGA(KK)
  280 CCNTINUE
  150 CCNTINUE
  210 CONTINUE
C
C         INSRRTION OF BOUNDARY POINTS
      DO 285 I=1,NUPBP
      II=IUPBP(I)
      DO 281 J=1,NUMSNP
      BIGC(J,II)=0.
      BIGC(II,J)=0.
  281 CCNTINUE
  285 CCNTINUE
      DO 311 I=1,NUMSNP
      DO 310 J=1,NUMSNP
      BIGCC(I,J)=0.
      PART(I,J)=.0
      DO 300 K=1,NUMSNP
      BIGCC(I,J)=BIGCC(I,J)+(BIGAB(K,J)- BIGC(K,J))*BIGA(I,K)
      PART(I,J)=PART(I,J)+(BIGAB(K,J)-2.*BIGC(K,J))*BIGA(I,K)
  300 CONTINUE
  310 CONTINUE
  311 CONTINUE
C
   05 CONTINUE
C
      IF(IND.EQ.1) GO TO 205
C
      DO 100 K=1,NUMSNP
```

```
      DPSI(K)=0.
C
      IF(K.GT.NUMEQ) GO TO 100
C
      DO 80 I=1,NUMSNP
      DPSI(K)=DPSI(K)+BIGCC(K,I)*PV(I)
   80 CCNTINUE
C
  100 CCNTINUE
C
      IF(T.GT.0.0) RETURN
      WRITE(6,60)T
   60 FORMAT(/2X,'FROM YVETTE',5X,'T=',G20.10)
      WRITE(6,65)(DPSI(K),K=1,NUMSNP)
      WRITE(6,65)(PSI(1,K),K=1,NUMSNP)
   65 FCRMAT(10(1X,E11.5))
C
      RETURN
C
  205 CCNTINUE
      DO 900 K=1,NUMSNP
      DO 880 L=1,NUMSNP
      KL=NUMSNP*(L-1)+K
      PW(KL)=PART(K,L)
  880 CCNTINUE
  900 CCNTINUE
C
      IF(T.GT.0.0) GO TO 950
      WRITE(6,65) (PW(I),I=1,1444)
      RETURN
C
  950 WRITE(6,700)
  700 FORMAT(//1X,'VALUES OF PW(IPT1),PW(IPT2),PW(IPT3)'/)
      WRITE(6,710) PW(IPT1),PW(IPT2),PW(IPT3)
  710 FCRMAT(5X,3G20.10)
C
      RETURN
      END
```

```
C
C
C
C
C
C       DATA PROCESSOR FOR FED2 PROGRAMS.
C
C
C
C
C
C    THIS PROGRAM TAKES THE STORED DATA SETS AND PRESENTS A
C    DIFFERENT METHOD AND/OR DISTURBANCE.  THE METHODS MUST
C    BE IN THE ORDER GIVEN. ALL METHODS FOR ONE DISTURBANCE
C    ARE RUN TOGETHER.  THE ORDER FOR DISTURBANCE TYPES IS
C    1-CENTRAL, 2-UNFIROM, 3-SKEW.  THE PROGRAM IS EASILY
C    MODIFIED TO ACCEPT CHANGES, EG. THE EXCLUSION OF A
C    DATA SET OR METHOD,OR THE EXAMINATION OF A DIFFERENT
C    TIME INTERVAL.
C
C          THE METHODS ARE
C                    EXACT       N**3
C                    LINEARIZED    N*N
C                    CRANK-NICOLSON
C                    GEAR    COMPACTED
C                    ADAMS   COMPACTED
C
C
C       EXPLANATION OF ITEMS.
C
C    NIT IS THE NUMBER OF TIME POINTS CONTAINED IN A GIVEN
C    DSET.  EACH TIME A DSET IS READ IN, ITS CORRESPONDING
C    NIT MUST BE READ IN FROM A CARD.
C
C    JJ IS THE NUMBER OF DIFFERENT DISTURBANCES EXAMINED.
C
C    JTYPE IS THE NUMBER OF DIFFERENT METHODS.
C
C    TCUT IS THE LAST NORMALIZED TIME POINT.
C
C    TSTRT IS THE APPROXIMATE FIRST NORMALIZED TIME POINT.
C
C    NUM IS THE NUMBER OF NORMALIZED TIME POINTS.
C
C
C       HOUSKEEPING.
C
      DIMENSION  FLUX(200),FTIM(200)
      DIMENSION STUF(50),T(50),HOLD(400,4),DSET(400,4)
      DIMENSION  EXACT(50,3),XLIN(50,3),ADAM(50,3),
     1   CRNK(50,3),GEER(50,3)
      DIMENSION  DIFX(50,3),DIFA(50,3),DIFG(50,3),DIFC(50,3)
      DIMENSION P(50)
      DIMENSION H(1800)
      DIMENSION STOR(50),ITB(12),RTB(28)
C
      DATA  ITB,RTB/12*0,28*.0/
      DATA CTR,UNF,SKW/'CENR','UNIF','SKEW'/
      DATA CTRL,CORE,REFL/'CTRL','CORE','REFL'/
      DATA CR/'CRNK'/
      DATA GE/'GEAR'/
      DATA AD/'ADAM'/
      DATA EX/'EXCT'/
      DATA XL/'XLIN'/
      DATA PT/'....'/
C
  110 FORMAT (5X,I5)
  205 FORMAT ('1',10X,'SINGLE CENTRAL DISTURBANCE'/)
  207 FORMAT ('1',10X,'UNIFORM DISTURBANCE'//)
  209 FORMAT ('1',10X,'SKEW DISTURBANCE'//)
  211 FORMAT (10X,'EXACT'//)
  212 FORMAT (10X,'XLIN'//)
  213 FORMAT (10X,'ADAM'//)
```

```
  214 FCRMAT (10X,'GEAR'//)
  215 FORMAT (10X,'CRNK'//)
  222 FORMAT(2X,'TIME',10X,'PSI(1)',8X,'PSI(13)',9X,
     1     'PSI(25)',8X,
     2    'TIME',10X,'PSI(1)',8X,'PSI(13)',9X,'PSI(25)'/)
  224 FORMAT (   2X,4(2X,1PE12.4))
  232 FORMAT (/10X,'PSI(1)    CENTER POINT'/)
  234 FCRMAT (/10X,'PSI(13)   CORE POINT'/)
  236 FORMAT (/10X,'PSI(25)   REFLECTOR POINT'/)
  238 FCRMAT (/10X,'VALUES COMPARED IN PERCENT DIFFERENCE.')
  240 FCRMAT (/10X,'NORMALIZED TIME VALUES'/)
  242 FORMAT (10X,'TIME',10X,'EXACT',10X,'LINEARIZED',10X,
     1     'GEER',10X,'ADAMS',8X,'CRANK-N'//)
  244 FORMAT ((7X,6(4X,E11.4)))
  246 FORMAT (7X,E11.4,4X,E11.4,08X,F6.2,10X,F6.2,9X,F6.2,
     1     8X,F6.6)
  500 FORMAT ('1'//10X,'     ')
C
C
C
C        ESTABLISH THE CONTROLING PARAMETERS.
C
      SCALE =1.E 14
      TSTRT=1.E-5
      NUM=50
      VAL=.05
      JTYPE=3
      JJ=3
C
C
C        ZERO THE STORAGE AREAS.
C
      DO 03 I=1,NUM
      DO 04 J=1,3
      EXACT (I,J)=.0
      XLIN  (I,J)=.0
      ACAM  (I,J)=.0
      GEER  (I,J)=.0
      CRNK  (I,J)=.0
      DIFX  (I,J)=.0
      DIFA  (I,J)=.0
      DIFG  (I,J)=.0
      DIFC  (I,J)=.0
   04 CONTINUE
   03 CCNTINUE
C
      DC 88 LL=1,JJ
C        LL DENOTES WHICH TYPE OF DISTURBANCE IS BEING
C        CONSIDERED.
C
C
C
C        CONSTRUCT THE NORMALIZED TIME INCREMENTS.
C
      XNUM=NUM
      TCUT=.1
      IF (LL .EQ. 2)  TCUT=.0329
      X=(TCUT/TSTRT)**(1./XNUM)
      DO 21 I=1,NUM
      T(I)=TSTRT*(X**I)
      P(I)= ALOG10(T(I))
   21 CONTINUE
C
      DO 86 LM=1,JTYPE
C
C        REAC AND WRITE ONE DATA SET IN THE RAW.
C
      READ (5,110) NIT
      READ (4) DSET
      IF (LL .EQ. 1) WRITE(6,205)
      IF (LL .EQ. 2) WRITE(6,207)
      IF (LL .EQ. 3) WRITE(6,209)
      IF (LM    .EQ. 1) WRITE(6,211)
```

```
      IF (LM     .EQ. 2) WRITE(6,212)
C     IF (LM     .EQ. 3) WRITE(6,213)
      IF (LM     .EQ. 3) WRITE(6,215)
      IF (LM     .EQ. 4) WRITE(6,214)
      IF (LM     .EQ. 5) WRITE(6,215)
      WRITE (6,224) ((DSET(I,K),K=1,4),I=1,NIT)
C
C
C        REBUILD THE DATA SET EXCLUDING TIME VALUES WHICH
C        ARE IDENTICAL.
C
      DO 06 I=1,4
      HOLD(1,I)= DSET(1,I)
   06 CONTINUE
C
      JK=1
      DO 07 I=2,NIT
      J=JK
      IF ((DSET(I,1)-HOLD(J,1))/HOLD(J,1) .LT. VAL) GO TO 07
      JK=JK+1
      DO 05 JIM=1,4
      HOLD(JK,JIM)=DSET(I,JIM)
   05 CONTINUE
   07 CONTINUE
C
C
      DO 17 M=2,4
C
C        SCREEN TO EXCLUDE DATA POINTS GOING NEGATIVE.
      DO 23 JEND=1,JK
      JDIF=JK-JEND + 1
      JP=JDIF+1
      IF (HOLD(JDIF,M) .LT. .0) GO TO 25
   23 CONTINUE
      JP=JDIF
      GO TO 26
   25 CONTINUE
      DO 26 I=1,JP
C        SMOOTH OUT ANY DISCONTINUITY.
      HOLD(I,M)=HOLD(JP,M)
   26 CONTINUE
C
      DO 27 I=1,NUM
      STUF(I)=P(I)
   27 CONTINUE
C
C
C        INSURE THE LAST TIME VALUE FOR DSET IS BEYOND THE
C        LAST NORMALIZED TIME POINT.
C
      JB=0
      DO 15  JF=1,JK
      JA=JK-JB
      JB=JB+1
      IF (HOLD(JA,1) .GE. T(NUM))  GO TO 19
      DO 13 I=2,4
      HOLD(JA,I)=SCALE
   13 CONTINUE
   15 CONTINUE
   19 CONTINUE
C
C
C
C        SCALE THE INPUT VALUES FOR THE INTERPOLATION ROUTIN
C
      DO 30 I= 1,JK
      FTIM(I)=ALOG10(HOLD(I,1))
      FLUX(I)=       (HOLD(I,M))/SCALE
   30 CONTINUE
C     WRITE (6,224) ((HOLD(I,K),K=1,4),I=1,JK)
C
C
```

```
C        NORMALIZE THE DATA SET
C
      JER=0
      JT=JK
      CALL ICS1VU (FLUX,FTIM,JT,NUM,STUF,H,JER)
C
C
C        TRANSFER NORMALIZED DSET TO APPROPRIATE CLASS AND
C        COMPUTE THE DIFFERENCE.
C
      L=M-1
C        L ESTABLISHES WHICH POINT IN THE REACTOR IS BEING
C        EXAMINED.
      GO TO (41, 43, 45, 47, 49),LM
   41 CONTINUE
      DO 42 I=1,NUM
      EXACT(I,L)=STUF(I)
   42 CONTINUE
      GO TO 51
   43 CONTINUE
      DO 44 I=1,NUM
      DIFX(I,L)=((STUF(I) -EXACT(I,L))/EXACT(I,L))*100.
      XLIN(I,L)=STUF(I)   *SCALE
   44 CONTINUE
      GO TO 51
   45 CONTINUE
      DO 46 I=1,NUM
      CRNK(I,L)=STUF(I)   *SCALE
      DIFC(I,L)=((STUF(I) -EXACT(I,L))/EXACT(I,L))*100.
   46 CONTINUE
      GO TO 51
   47 CONTINUE
      DO 48 I=1,NUM
      DIFG(I,L)=((STUF(I) -EXACT(I,L))/EXACT(I,L))*100.
      GEER(I,L)=STUF(I)   *SCALE
   48 CONTINUE
      GO TO 51
   49 CONTINUE
      DO 50 I=1,NUM
      DIFA(I,L)=((STUF(I) -EXACT(I,L))/EXACT(I,L))*100.
      ADAM(I,L)=STUF(I)   *SCALE
   50 CONTINUE
   51 CONTINUE
   17 CONTINUE
   86 CONTINUE
C
      DO 63 L=1,3
      DO 61 I=1,NUM
      EXACT(I,L)=EXACT(I,L)*SCALE
   61 CONTINUE
   63 CONTINUE
C
C
C
C
C        WRITE OUT RESULTS AND DRAW THE GRAPHS.
      DO 57 LCNT=1,4
      DO 58  L =1,3
      IF (LL .EQ. 1) WRITE(6,205)
      IF (LL .EQ. 2) WRITE(6,207)
      IF (LL .EQ. 3) WRITE(6,209)
C
      WRITE (6,240)
      IF(L  .EQ. 1) WRITE(6,232)
      IF(L  .EQ. 2) WRITE(6,234)
      IF(L  .EQ. 3) WRITE(6,236)
      IF (LCNT .EQ. 2) WRITE(6,238)
C
      WRITE(6,242)
      IF (LCNT .EQ.1) WRITE(6,244)
     1   (T(I),EXACT(I,L),XLIN(I,L),GEER(I,L),ADAM(I,L),CRNK
     2   I=1,NUM)
```

```
      IF(LCNT .EQ. 2) WRITE(6,246)
     1   (T(I),EXACT(I,L),DIFX(I,L),DIFG(I,L),DIFA(I,L),DIFC
     2   I=1,NUM)
      LPLT=LPLT+1
      IF (LCNT .LE. 2) GO TO 54
C
C    INSERT CALLS TO PLOTTING ROUTINES.
C
      RTB(6)=PT
      RTB(2)=.0
      IF (LCNT .EQ. 4) RTB(2)=5.
      IF  (LL .EQ. 1)  RTB(5)  =CTR
      IF  (LL .EQ. 2)  RTB(5)  =UNF
      IF  (LL .EQ. 3)  RTB(5) = SKW
      IF (L .EQ. 1) RTB(7)    =CTRL
      IF (L .EQ. 2) RTB(7)    =CORE
      IF (L .EQ. 3) RTB(7)    =REFL
C
      DO 97 IPLOT=1,JTYPE
      GO TO (71,73,75,77,79),IPLOT
   71 CONTINUE
      RTB(3)   =EX
      ITB(1) =1
      DO 72 I=1,NUM
      STUF(I) = EXACT(I,L)
      IF (LCNT .EQ. 4) STUF(I)=0.
   72 CONTINUE
      GO TO 95
   73 CONTINUE
      RTB(3)   =XL
      ITB(1) = 2
      DO 74 I=1,NUM
      STUF(I)=XLIN(I,L)
      IF (LCNT .EQ. 4) STUF(I)=DIFX(I,L)
   74 CONTINUE
      GO TO 95
   75 CONTINUE
      RTB(3)   = CR
      ITB(1)=3
      DO 76 I=1,NUM
      STUF(I)=CRNK(I,L)
      IF (LCNT .EQ. 4) STUF(I)=DIFC(I,L)
   76 CONTINUE
      GO TO 95
   77 CONTINUE
      RTB(3)   = GE
      DO 78  I=1,NUM
      STUF(I)=GEER(I,L)
      IF (LCNT .EQ. 4) STUF(I)=DIFG(I,L)
   78 CONTINUE
      GO TO 95
   79 CONTINUE
      RTB(3)   = AD
      DO 80 I=1,NUM
      STUF(I) = ADAM(I,L)
      IF (LCNT .EQ. 4) STUF(I)=DIFA(I,L)
   80 CONTINUE
   95 CONTINUE
      CALL DRAWP(NUM,P,STUF,ITB,RTB)
   97 CONTINUE
C
   54 CONTINUE
   58 CONTINUE
   57 CONTINUE
C
C
   88 CONTINUE
C
      WRITE (6,500)
      STOP
      END
```

```
C
C
C                 A PROGRAM TO TEST THE COMPACT METHCD
C                 OF FINITE ELEMENT MATRIX WORK.
C
C
C
C
C
C
C    INPUT INFORMATION
C
C    DATA CARDS ARE REQUIRED IN THE FOLLOWING ORDER,
C
C    NODY, NUMSNP  (NR NODES IN Y DIRECTION, NR SYSTEM
C   NCDAL POINTS)
C
C    PSI   THE SET OF VALUES FOR TEST MULTIPLICATION.
C
C    NUMEL    NR ELEMENTS IN THE MESH
C
C    ELNOD    THE ELEMENT CONNECTIVITY MATRIX.
C
C
C
      INTEGER*4 ELNOD
      DIMENSION VALUE(50),MNOD(50,7),VMATX(50,7),
     1   BIGA(50,50),ELNOD(75,3)
      DIMENSION  PSI(50), COMPCT(50),USUAL(50)
      DIMENSION AA(3,3)
  110 FORMAT (I15,5X,I5)
  119 FORMAT ('1'/////////'     ')
  120 FCRMAT (F9.2,6F10.2)
  150 FORMAT (5X,3I10)
C
  210 FORMAT('1',10X,'NODE NEIGHBOR CONNECTIVITY'//)
  212 FORMAT (10X,'NODE',5X,'K2',5X,'K3',5X,'K4',5X,'K5',
     1    5X,'K6',5X,'K7'//)
  215 FORMAT (10X,7(I5,2X)/)
  220 FCRMAT('1'///10X,'THIS IS THE NUMSNP X 7 MATRIX'//)
  225 FORMAT (10X,7(F6.0,1X)/)
  230 FORMAT ('1'///10X,'THE ELEMENT CONNECTIVITY MATRIX'/)
  235 FCRMAT (10X,I4,3(3X,I4))
  250 FORMAT('1'///10X,'THIS IS THE USUAL NUMSNP X NUMSNP ',
     1    'BIGA MATRIX'//)
  255 FORMAT (2X,20(F5.0,1X)/)
  270 FORMAT ('1'///10X,'COMPACT FORM',5X,'USUAL FORM'//)
  272 FORMAT (5X,I2,7X,F8.0,07X,F8.0)
C
C
      READ(5,110) NODY,NUMSNP
      READ (5,120) (PSI  (I),I=1,NUMSNP)
      WRITE (6,119)
      WRITE (6,120) (PSI  (I), I=1,NUMSNP)
C
C    CONSTRUCT THE NEIGHBOR NODES OF THE HEXAGON.
      WRITE (6,210)
      WRITE (6,212)
C
      DO 35 K1=1,NUMSNP
      N=1
    5 CONTINUE
      NN=N*NODY
      N=N+1
      IF ((K1-NN) .GT. 0)  GO TO 5
      K3=K1-1
      K6=K1+1
      K2=K1-NCDY
      K7=K2+1
      K5=K1+NODY
      K4=K5-1
      IF(K1 .GT. NODY)  GO TO 6
      K2=0
      K7=0
```

```
    6 IF(K1 .NE. NN)  GO TO 7
      K6=0
      K7=0
    7 IF(K1 .LE. (NUMSNP-NODY))  GO TO 8
      K5=0
      K4=0
    8 IF (K1 .NE. (NN-NODY+1))  GO TO 9
      K3=0
      K4=0
    9 CONTINUE
C
      MNOD(K1,1)=K1
      MNOD(K1,2)=K2
      MNOD(K1,3)=K3
      MNOD(K1,4)=K4
      MNOD(K1,5)=K5
      MNOD(K1,6)=K6
      MNOD(K1,7)=K7
      WRITE (6,215)  (MNOD(K1,I),I=1,7)
   35 CONTINUE
C
C
C
C    READ THE CONNECTIVITY MATRIX
      READ (5,110) NUMEL
      WRITE (6,230)
      DO 60  I=1,NUMEL
      READ (5,150) (ELNOD(I,J),J=1,3)
      WRITE (6,235)  I,(ELNOD(I,J),J=1,3)
   60 CONTINUE
C
C     ZERO THE BIGA, COMPCT, AND VALUE MATRICIES.
      DO 62 I=1,NUMSNP
      VALUE(I)=0.
      DO 61 J=1,NUMSNP
      VMATX(I,J)=0.
      BIGA(I,J)=0.
   61 CONTINUE
   62 CONTINUE
C
C
C
C
C     CONSTRUCT THE N X 7 MATRIX
C
      DO 16 L=1,NUMEL
      Y=FLOAT(L)
      R1=2.*Y
      R2=3.*Y
      R3=4.*Y
      AA(1,1)=2.*R1+R2+2.*R3
      AA(1,2)=R1+R2+2.*R3
      AA(1,3)=2.*R1+2.*R2+R3
      AA(2,1)=2.*R1+2.*R2+2.*R3
      AA(2,2)=R1+R2+R3
      AA(2,3)=R1+2.*R2+2.*R3
      AA(3,1)=AA(1,1)
      AA(3,3)=AA(2,3)
      AA(3,2)=AA(2,1)
      DO 15 K=1,3
      KK=ELNOD(L,K)
      DO 14 I=1,3
      II=ELNOD(L,I)
C
C     SEARCH KK ROW FOR NODE II.  IF FOUND, POST THE VALUE
C     TO VMATX(KK,M).  OTHERWISE, CONTINUE STEPPING
C     THROUGH THE ELEMENT.
      DO 12 M=1,7
      NOW=M
      MM   =MNOD(KK,M)
      IF (MM    .EQ. II) GO TO 13
   12 CONTINUE
```

```
   13 CONTINUE
      VMATX(KK,NOW)=VMATX(KK,NOW) + AA(K,I)
   14 CONTINUE
   15 CONTINUE
   16 CONTINUE
C
C     WRITE OUT THE N X 7
      WRITE (6,220)
      WRITE (6,212)
      DO 37 K1=1,NUMSNP
      WRITE (6,225) (VMATX(K1,J),J=1,7)
   37 CONTINUE
C
C
C
C     GENERATE THE ELEMENT VALUES AND MOVE THEM INTO THE NXN
C
      DO 68 L=1,NUMEL
      Y=FLOAT(L)
      R1=2.*Y
      R2=3.*Y
      R3=4.*Y
      AA(1,1)=2.*R1+R2+2.*R3
      AA(1,2)=R1+R2+2.*R3
      AA(1,3)=2.*R1+2.*R2+R3
      AA(2,1)=2.*R1+2.*R2+2.*R3
      AA(2,2)=R1+R2+R3
      AA(2,3)=R1+2.*R2+2.*R3
      AA(3,1)=AA(1,1)
      AA(3,2)=AA(2,1)
      AA(3,3)=AA(2,3)
      DO 67 K=1,3
      KK= ELNOD(L,K)
      DO 66 I=1,3
      II=ELNOD(L,I)
      BIGA(KK,II)=BIGA(KK,II)+AA(K,I)
   66 CONTINUE
   67 CONTINUE
   68 CONTINUE
C
C     WRITE OUT THE N X N.
      WRITE (6,250)
      DO 75 I=1,NUMSNP
      WRITE (6,255) (BIGA(I,J), J=1,NUMSNP)
   75 CONTINUE
C
C
C
C     PERFORM A SIMPLE MATRIX X COLM MULTIPLICATION ON BOTH
C     AND COMPARE THE RESULTS.
C
      DO 72 I=1,NUMSNP
      COMPCT(I)=0.
      DO 71 J=1,7
      NAME=MNOD(I,J)
       COMPCT(I)=PSI(NAME)*VMATX(I,J)+ COMPCT(I)
   71 CONTINUE
   72 CONTINUE
C
      DO 42 I=1,NUMSNP
      USUAL(I)=0.
      DO 41 J=1,NUMSNP
      USUAL(I)=BIGA(I,J)*PSI(J)+USUAL(I)
   41 CONTINUE
   42 CONTINUE
C
C
      WRITE(6,270)
      DO 80 I=1,NUMSNP
      WRITE(6,272) I, COMPCT(I),USUAL(I)
   80 CONTINUE
C
```

```
      STOP
      END
//GO.SYSIN DD *
                4                  20
 8.        10.        20.        30.        40.        50.        60.
85.        15.        25.        35.        45.        55.        65.
87.        17.        27.        37.        47.        57.
               24
1               1               5               2
 2              5               6               2
 3              6               3               2
 4              6               7               3
 5              7               4               3
 6              7               8               4
 7              9               6               5
 8              9              10               6
 9             10               7               6
10             10              11               7
11             11               8               7
12             11              12               8
13             13              10               9
14             13              14              10
15             14              11              10
16             14              15              11
17             15              12              11
18             15              16              12
19             17              14              13
20             17              18              14
21             18              15              14
22             18              19              15
23             19              16              15
24             19              20              16
```

	COMPACT FORM	USUAL FORM
1	780.	780.
2	8130.	8130.
3	21120.	21120.
4	25810.	25810.
5	15799.	15799.
6	54965.	54965.
7	98365.	98365.
8	72195.	72195.
9	39740.	39740.
10	126275.	126275.
11	195386.	195386.
12	113725.	113725.
13	92605.	92605.
14	241556.	241556.
15	288460.	288460.
16	146967.	146967.
17	74300.	74300.
18	144144.	144144.
19	144310.	144310.
20	39144.	39144.

THE ELEMENT CONNECTIVITY MATRIX

1	1	5	2
2	5	6	2
3	6	3	2
4	6	7	3
5	7	4	3
6	7	8	4
7	9	6	5
8	9	10	6
9	10	7	6
10	10	11	7
11	11	8	7
12	11	12	8
13	13	10	9
14	13	14	10
15	14	11	10
16	14	15	11
17	15	12	11
18	15	16	12
19	17	14	13
20	17	18	14
21	18	15	14
22	18	19	15
23	19	16	15
24	19	20	16

THIS IS THE NUMSNP X 7 MATRIX

NODE	K2	K3	K4	K5	K6	K7
15.	0.	0.	0.	13.	14.	0.
96.	0.	15.	48.	81.	54.	0.
171.	0.	48.	114.	147.	90.	0.
141.	0.	80.	180.	108.	0.	0.
151.	18.	0.	0.	105.	152.	44.
458.	74.	148.	246.	279.	214.	95.
618.	134.	216.	312.	345.	276.	149.
345.	96.	284.	378.	216.	0.	0.
433.	98.	0.	0.	195.	338.	203.
938.	254.	352.	444.	477.	400.	257.
1098.	314.	420.	510.	543.	462.	311.
549.	192.	488.	576.	324.	0.	0.
709.	182.	0.	0.	285.	524.	365.
1418.	434.	556.	642.	675.	586.	419.
1578.	494.	624.	708.	741.	648.	473.
753.	288.	692.	774.	432.	0.	0.
585.	266.	0.	0.	0.	260.	527.
825.	614.	360.	0.	0.	286.	581.
903.	674.	396.	0.	0.	312.	635.
216.	384.	432.	0.	0.	0.	0.

NODE NEIGHBOR CONNECTIVITY

NODE	K2	K3	K4	K5	K6	K7
1	0	0	0	5	2	0
2	0	1	5	6	3	0
3	0	2	6	7	4	0
4	0	3	7	8	0	0
5	1	0	0	9	6	2
6	2	5	9	10	7	3
7	3	6	10	11	8	4
8	4	7	11	12	0	0
9	5	0	0	13	10	6
10	6	9	13	14	11	7
11	7	10	14	15	12	8
12	8	11	15	16	0	0
13	9	0	0	17	14	10
14	10	13	17	18	15	11
15	11	14	18	19	16	12
16	12	15	19	20	0	0
17	13	0	0	0	18	14
18	14	17	0	0	19	15
19	15	18	0	0	20	16
20	16	19	0	0	0	0

THIS IS THE USUAL NUMSNP X NUMSNP BIGA MATRIX

15.	14.	0.	0.	13.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
15.	96.	54.	0.	48.	81.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	48.	171.	90.	0.	114.	147.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	80.	141.	0.	0.	180.	108.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
18.	44.	0.	0.	151.	152.	0.	0.	105.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	74.	95.	0.	148.	458.	214.	0.	246.	279.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	134.	149.	0.	216.	618.	276.	0.	312.	345.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	96.	0.	0.	284.	345.	0.	0.	378.	216.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	98.	203.	0.	0.	433.	338.	0.	0.	195.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	254.	257.	0.	352.	938.	400.	0.	444.	477.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	314.	311.	0.	420.	1098.	462.	0.	510.	543.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	192.	0.	0.	488.	549.	0.	0.	576.	324.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	182.	365.	0.	0.	709.	524.	0.	0.	285.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	434.	419.	0.	556.	1418.	586.	0.	642.	675.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	494.	473.	0.	624.	1578.	648.	0.	708.	741.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	288.	0.	0.	692.	753.	0.	0.	774.	432.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	266.	527.	0.	0.	585.	260.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	614.	581.	0.	360.	825.	286.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	674.	635.	0.	396.	903.	312.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	384.	0.	0.	432.	216.

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c.1 Effective methods
for solution of non-
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